

Lecture Notes Math 104: Calculus I (Spring
2007)

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1 TALK SLOWLY AND WRITE NEATLY AND BIG!!

2 Introduction

Before Class

- (1) Write my name, office and office hours on the board.
- (2) Write name of teaching assistant.

– Dan Pomerleano (dpomerle@math.upenn.edu)

- (3) Write the course web address

(www.math.upenn.edu/nate/teaching/2007/spring/math_104/index.html)

Also write my homepage address

(www.math.upenn.edu/nate/index.html)

- (1) Introduce myself and tell about my office hours (Wednesdays 2:15 pm - 3:15 pm).
- (2) Introduce the TA

- (3) Explain that we are one section of 4. There are up to 400 people taking the class.
- (4) The way grading will work is as follows
- Homework will be worth 25%. You will get 25% credit if you just finish all the problems. If you finish more than half you will get 12.5%. Three problems are then chosen randomly and graded.

Homework is a crucial part of learning calculus. You must spend a great time practicing the skills you are learning before they are mastered.

The homeworks are posted on the website. They are due at the beginning of section. There may be minor modifications to the collection throughout the semester depending on how things are going

so check back often. But I will announce in class any time there is a change.

It is strongly encouraged that you work in groups. However you must write up your solutions yourself and put on your homework the names of everyone in your group with whom you have worked.

- There will be 3 midterms worth 15% of your grade each and one final worth 30% of your grade

These will be taken by everyone in math 104.

The days/times of the midterms are

- * Wednesday February 7 5-50pm.
- * Wednesday March 14 5-50pm.
- * Wednesday April 11 5-50pm.

- Final Grades. The grades of the entire math 104 class will be curved. 30% of the class will get an A, 30% will get a B, 30% will get a C and 10% will get a D or F.

3 Area Between Curves

Draw several copies of a picture of two curves

Say we want to approximate the area between these two curves where $f(x) \geq g(x)$.

Draw 3 rectangles with midpoints on the first picture (make sure to label the x points).

Draw 6 rectangles with midpoints on the second picture (make sure to label the x points).

Definition 3.0.1. We see that the area is

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n [f(x_i) - g(x_i)] \Delta x$$

Or in the notation of calculus we have

$$\int_a^b [f(x) - g(x)] dx$$

Example 1

Lets look at an example. What is the area between

$$f(x) = 4x, g(x) = x^2$$

Well first off lets graph these two functions.

Graph the two functions

Shade area we want to find

Observe

we want to find the area between $f(x)$, $g(x)$ with bounds the points of intersections (draw dotted lines for the points).

We see that the

$$f(x) = g(x) \Rightarrow x = 0 \text{ or } x = 2$$

We also see that in this interval $f(x) \geq g(x)$ so the area between the curves is

$$\int_0^2 (4x - x^2) - \left[\frac{1}{3}x^3 \right]_0^2 = (2 \times 2^2 - \frac{1}{3} \times 2^3) - (2 \times 0 - \frac{1}{3} \times 0) = 10 - \frac{8}{3} = \frac{22}{3}$$

Example 2

Now let's look at an easy example.

Let's look at the area between

$$f(x) = \sin(x) + 1 \text{ and } g(x) = 1, -\pi \leq x \leq \pi$$

The first thing we want to do is graph this

Graph this

Now we see we have run into a problem. We no longer have $f(x) \geq g(x)$ or $g(x) \geq f(x)$ over the entire interval. And if we try and just blindly plug in the previous formula we would get

$$\text{Area} = \int_{-\pi}^{\pi} (\sin(x) + 1) - 1 = 0$$

WHICH IS WRONG!!

What we have to do is break this graph into intervals.

Label regions Area A, Area B

We then

see that the area between these curves is

$$\begin{aligned}
 \text{Area} &= \text{Area}_A = \text{Area}_B = \int_0^\pi f(x) - g(x) dx + \int_{-\pi}^0 g(x) - f(x) dx \\
 &= \int_0^\pi (\sin(x) + 1) - 1 dx + \int_{-\pi}^0 1 - (\sin(x) + 1) dx \\
 &\quad (*) = \int_0^\pi \sin(x) dx + \int_{-\pi}^0 -\sin(x) dx \\
 &\quad = -\cos(x)]_0^\pi + \cos(x)]_{-\pi}^0 \\
 &\quad = -(-1 - 1) + (1 - (-1)) = 4
 \end{aligned}$$

However (*) is also just

$$= \int_{-\pi}^\pi |(\sin(x) + 1) - 1| dx$$

And in fact if we think about it, this is the general form for the area between two curves.

Theorem 3.0.2. *Let $f(x), g(x)$ be two continuous functions. Then the area between $f(x), g(x)$ in the interval (a, b) is*

$$\int_a^b |f(x) - g(x)| dx$$

4 Vertical Area

Sometimes calculating the horizontal area doesn't work.

Draw picture Observe that if we try and use the previous method we run into a problem.

Draw the vertical rectangles and show how they intersect the curve.

But we can sometimes instead use horizontal rectangles. (Draw them).

Here if we right x_L for the left boundary and x_R for the right boundary then we see that the area is

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n [x_R - x_L] \Delta y$$

Or in the notation of calculus we have

$$\int_d^c [x_R - x_L] dy = \int_d^c [f(y) - f(y)] dy$$

Example

Lets calculate the area between

$$y^2 = x \text{ and } 2y^2 = x + 1$$

DRAW

The first thing we need to do is find the points of intersection. We see these occur when $y^2 = 2y^2 - 1 = x$ or when $x = y^2 = 1$, i.e. $(1, 1), (1, -1)$.

So we want our bounds of integration to be from -1 to 1 and our integral will be

$$\begin{aligned} \int_{-1}^1 (y^2 - (2y^2 - 1)) dy &= \int_{-1}^1 (1 - y^2) dy \\ &= (y - 1/3y^3) \Big|_{-1}^1 = (1 - 1/3) - (-1 + 1/3) = 4/3 \end{aligned}$$

Now we can ask when does this formula work. Well this formula work when the rectangles we drew don't intersect the curve

Draw example of where it doesn't work

However we see that these rectangles don't intersect the curve if and only if we can express y as a function of x on each of the borders. That is to each value y there is at most one value x for which (x, y) is on the curve.

5 Examples

If there is still time go through some more examples