Math 151 Practice Midterm 2:

April 15, 2010

Rules: Write all answers in the spaces provided below. Do NOT separate the sheets. No notes or calculators allowed. All problems are worth the same amount. Make sure that in each problem you have at least one sentence explaining the key idea. Good Luck!

Name: _____

Problem 1: Find two integers x and y such that $GCD(99, 147) = x \cdot 99 + y \cdot 147$.

Problem 2: Consider the following statement "There are two natural numbers, $a > 1$ and $b > 1$ with $a \neq b$ such that $ab = x^2$". Characterize those values of x for which this is true.

Problem 3: Find a shape S such that for every x in the plane there is a dilation D such that $x \in D(S)$.

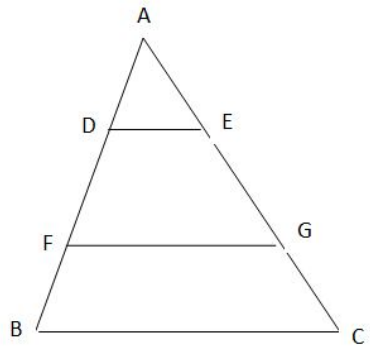
Problem 4: We say a transformation F of the plane is "idempotent" if $F \circ F = F$. For each of the following, either prove it true, or give a counterexample:

- There are no bijective idempotents other than the identity.
- There are in injective idempotents other than the identity.
- There are no surjective idempotents other than the identity.

Problem 5: Suppose $ABCD$ is a parallelogram with AB and CD opposite sides. If E is the midpoint of AB and F is the midpoint of CD show that $AB||EF$ and $EF||CD$.

Problem 6: Suppose D is **any** transformation of the plane. Consider the following statement: "If $D(S)$ is not convex then S is not convex." Is this always true? If yes prove it. If no give a counterexample.

Problem 7: Consider the triangle



Recall that a trapazoid is a quadrilateral which has only one pair of parallel sides. Prove that for triangles as in the figure the following are equivalent:

- $FGCB$ and $DEGF$ are both trapazoids
- $\Delta ADE \sim \Delta AFG$ and $\Delta ADE \sim \Delta ABC$

Problem 8: Suppose P and Q are distinct points in the plane and let D_P, D_Q be two dilations with centers P and Q respectively where the scale factor of D_P is $1/2$ and the scale factor of D_Q is 2. Show that $D_P \circ D_Q$ has no fixed points. I.e. that $(\forall X) D_P \circ D_Q(X) \neq X$ (and in particular this implies $D_P \circ D_Q$ is not a dilation).