Math 151 Practice Final:

May 12, 2010

Rules: Write all answers in the spaces provided below. Do NOT separate the sheets. No notes or calculators allowed. All problems are worth the same amount. Make sure that in each problem you have at least one sentence explaining the key idea. Good Luck!

Name: _____

Problem 1: Express the following as a fraction in lowest terms:

$$(1 + 2^*)(3 + 4^*) + 5$$

Problem 2: Assume $y > x > 0$. List all statements of the form $A \leq B$ or $A < B$ which hold for all values of x and y when A and B are among the following:

(a) $\frac{1}{x}$.

(b) $\frac{1}{y}$.

(c) $\frac{y}{y-x}$.

(d) $\frac{y-x}{y}$.

(e) 1.

Prove all statements.

Problem 3: Suppose Alice and Bob live 100 miles away from each other and suppose they start driving towards each other, each at a constant rate. Further suppose Alice drives 50% faster than Bob. If they meet after two hours how fast was Bob driving? How fast was Alice driving?

Problem 4: Express each as a fraction in lowest terms:

(a) $(1.1) \cdot (1.2) \cdot (1.3)$

(b) $\dfrac{1.1}{\frac{1.2}{1.3}}$

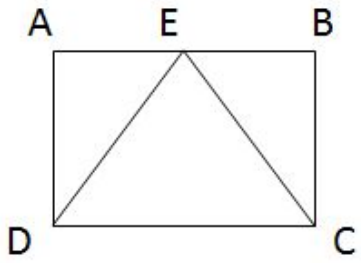
(c) $1 + \dfrac{1.1}{1 + \dfrac{1.2}{1 + \dfrac{1}{1.3}}}$

Problem 5: Find all values of x such that

$$6 - 3 \cdot |x - 2| > 4$$

Problem 6: What is $GCD(408, 262)$?

Problem 7: Consider the following picture



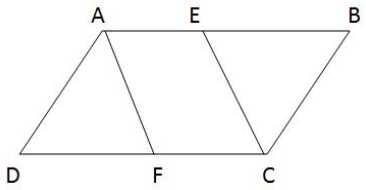
where

- $ABCD$ is a rectangle.
- E is the midpoint of AB.

Prove $|\angle EDC| = |\angle ECD|$.

Problem 8: Suppose ΔABC is a right triangle with $|\angle ABC| = 90^o$. Show that the area of ΔABC is $\frac{1}{2} \cdot |AB| \cdot |BC|$

Problem 9: Consider the following picture



where

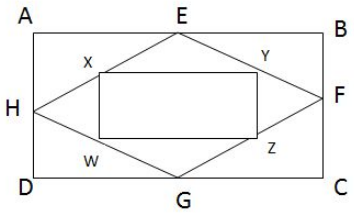
$ABCD$ is a parallelogram.

$E \in AB$ and $F \in CD$.

$AE||FC$.

Prove that $\Delta ADF \cong \Delta CBE$.

Problem 10: Consider the following picture



where

$ABCD$ is a rectangle.

E is the midpoint of AB, F is the midpoint of BC, G is the midpoint of CD and H is the midpoint of AD.

X is the midpoint of HE, Y is the midpoint of EF, Z is the midpoint of FG and W is the midpoint of GH.

Prove that $XY||AB$ and that $XYZW$ is a rectangle.

Problem 11: Consider the following statement:

(a) Suppose T is a transformation of the plane such that whenever C is non-convex, $T(C)$ is non-convex. Then T is injective.

Either prove it is true or find a counterexample.

Problem 12: Consider the transformation of the plane $T(x, y) = ((x + y)^3, (x - y)^3)$. Is this transformation injective? Surjective? Bijective?

Problem 13: Suppose $P = (4, 4)$ and $Q = (5, 6)$. What is the slope of L_{PQ}? What is the x-intercept of L_{PQ}? What is the y-intercept of L_{PQ}? What is an equation of the graph of L_{PQ}?

Problem 14: Find all solutions to the system of equations:

(1) $4x - 2y = 3$

(2) $x + y = 4$

Problem 15: Find all solutions to the system of equations:

(1) $4x - 2y = 4$

(2) $2x - y = 2$

Problem 16: Simplify the following as much as possible:

$$x + \cfrac{x}{x + \cfrac{x}{x + \cfrac{x}{x + \cfrac{x}{x}}}}$$

Problem 17: List the following from smallest to largest

(a) $5^{19} - 2^{19}$

(b) $5^{18} + 5^{17} + \cdots + 5^3 + 5^2 + 5 + 1$

(b) $2^{18} + 2^{17} + \cdots + 2^3 + 2^2 + 2 + 1$

(b) 2^{19}