MATH 10A Aug 4, 2012

1) a) $\lim_{x\to 4} \frac{x^2-16}{x-4} = \lim_{x\to 4} \frac{(x+4)(x-4)}{x-4} = \lim_{x\to 4} (x+4) = 8$

b) limit does not exist as value of sine ocillate when $x \to \infty$.

c) $\lim_{x\to 0} \frac{x^4-3x+7}{2x^4-6} = \frac{7}{-6} = -\frac{7}{6}$

2) a) $P(t) = 156 \cdot 2^t$

b) mice / month

3) a) $f(3) = \frac{e^{3-2}}{3^3} = \frac{e}{27}$, $f(1) = \frac{e^{1-2}}{1^3} = e^{-1}$

av. rate of change $= \frac{f(3) - f(1)}{3-1}$

$= \frac{e/27 - e^{-1}}{2}$

b) $f'(x) = \frac{e^{x-2} \cdot x^3 - 3x^2 e^{x-2}}{x^6}$

$f'(2) = \frac{2^3 - 3 \cdot 2^2}{2^6} = \frac{2-3}{2^4} = \frac{-1}{16}$

$y = -\frac{1}{16}x + b$

$f(2) = \frac{1}{2^3} = \frac{1}{8}$

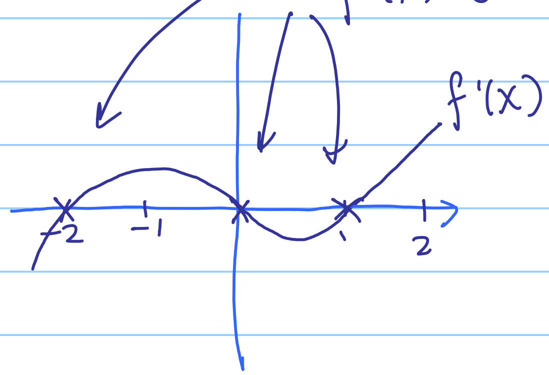
$\therefore \frac{1}{8} = -\frac{1}{16}(2) + b$

$b = \frac{1}{4}$

$\therefore$ tangent eqn: $y = -\frac{1}{16}x + \frac{1}{4}$

c) $f(3) \approx -\frac{1}{16}(3) + \frac{1}{4} = \frac{1}{16}$

4) $f'(x)=0$ at $x=-2,0,1$



5) a) $\frac{1}{1+(1-3x^2)^2}\cdot(-6x)$

b) $e^{\sin x+x}\left(\cos x+1\right)$

c) $\frac{3x^2}{9}\left(2\ln x-1\right)+\frac{x^3}{9}\left(\frac{2}{x}\right)$

6) $\cos(xy)\cdot\left(y+x\frac{dy}{dx}\right)=4y\frac{dy}{dx}$

$y\cos xy=\left(4y-x\cos(xy)\right)\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{y\cos xy}{4y-x\cos(xy)}$

7) $f'(x)=2xe^{-x}-x^2e^{-x}=(2x-x^2)e^{-x}$

$f'(x)=0$

$2x-x^2=0$

$x=0$ or $x=2$

$f''(x)=(2-2x)e^{-x}-(2x-x^2)e^{-x}$

$f''(0)=2>0$ $\therefore$ $x=0$ is local min

$f''(2)=-2e^{-2}<0$ $\therefore$ $x=2$ is local max

8)

$$f(x) = x^3 - 6x^2$$
$$f'(x) = 3x^2 - 12x$$

$$f'(x) = 0$$
$$3x(x-4) = 0 \Rightarrow x = 0 \text{ or } x = 4$$
$$f''(x) = 6x - 12$$
$$f''(0) = -12 < 0 \quad \therefore \text{local max}$$

$$f(-1) = (-1)^3 - 6 = -7$$
$$f(0) = 0$$
$$f(2) = 8 - 6 \cdot 4 = -16$$

$\therefore$ $x = 2$ is global min & $x = 0$ is global max over $[-1, 2]$

9)

$$\text{Surface area} = 2x^2 + 4xh = 54$$
$$h = \frac{54 - 2x^2}{4x} = \frac{27 - x^2}{2x}$$

Volumn function: $V(x) = x^2 h$
$$= x^2\left(\frac{27 - x^2}{2x}\right)$$
$$= \frac{1}{2}(27x - x^3)$$
$$V'(x) = \frac{1}{2}(27 - 3x^2)$$
$$V'(x) = 0$$
$$27 - 3x^2 = 0$$
$$x^2 = 9$$
$$x = 3$$
$$V''(x) = -\frac{3}{2}x^2$$
$$V''(3) < 0 \Rightarrow \max$$
$\therefore$ largest vol $= V(3) = \frac{1}{2}(27 \cdot 3 - 3^3) = 27$ cubic inches