10A Practice Midterm 2

1) $g(x) = x^{1/5}$

$$\lim_{h\to 0} \frac{g(0+h)-g(0)}{h} = \lim_{h\to 0} \frac{h^{1/5}}{h} = \lim_{h\to 0} \frac{1}{h^{4/5}}$$

limit does not exist.

$\therefore$ $g(x)$ is not differentiable at $x=0$

2) a) $\frac{d}{dx}\left(\frac{x^2-3}{4^x+1}\right) = \frac{2x(4^x+1)-(x^2-3)(4^x \ln 4)}{(4^x+1)^2}$

b) $\frac{d}{dx}\left(e^{x^6}\cos x + \pi\right) = 6x^5 e^{x^6}\cos x - e^{x^6}\sin x$

c) $\frac{d}{dx}\sqrt{2\ln x + 10} = \frac{1}{2}(2\ln x+10)^{-1/2}\left(\frac{2}{x}\right)$

$$= \frac{1}{x\sqrt{2\ln x+10}}$$

d) $\frac{d}{dx}\left(\arctan(3-x) + \tan(2x)\right)$

$$= \frac{1}{1+(3-x)^2}\cdot(-1) + \sec^2(2x)\cdot 2$$

3) $f(x) = \sin(x^2)$

$f'(x) = 2x\cos(x^2)$

$f''(x) = 2\cos(x^2) - 4x^2\sin(x^2)$

a) At $x=\sqrt{\pi}$, $f'(\sqrt{\pi}) = 2\sqrt{\pi}\cos(\pi) = -2\sqrt{\pi} < 0$

$\therefore$ f is decreasing at $\sqrt{\pi}$

b) $f''(\sqrt{\pi}) = 2\cos(\pi) - 4\pi\sin(\pi) = -2 < 0$

$\therefore$ f is concave down at $x=\sqrt{\pi}$

4) $x^2y - 2y + 5 = 0$

a) $2x + x^2\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{2x}{2 - x^2}$$

b) At $(2, -5/2)$, $\left.\frac{dy}{dx}\right|_{x=2} = \frac{4}{2-4} = -2$

$y = -2x + b$

$-5/2 = -2(2) + b \Rightarrow b = 3/2$

$y = -2x + 3/2$