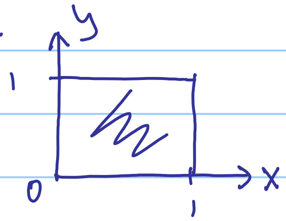
MATH 20E Week 5 Winter, 2013

§ 5.2

Q4 Compute the vol. of solid bdd by xz, yz, xy plane, $x=1$, $y=1$ & $z=x^2+y^4$.

Sol: On xy-plane

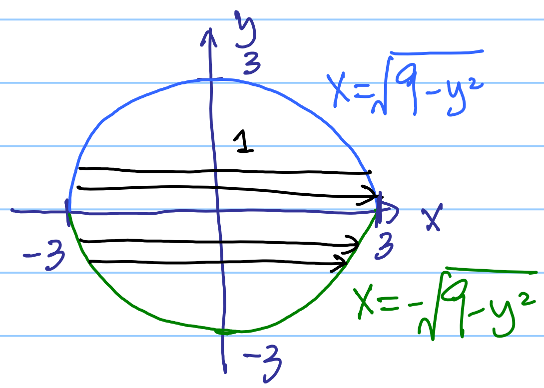


$$\therefore \quad \text{Vol} = \int_0^1 \int_0^1 (x^2+y^4)\,dx\,dy$$
$$= \int_0^1 \left[\frac{x^3}{3} + xy^4\right]_0^1 dy$$
$$= \int_0^1 \frac{1}{3} + y^4\,dy$$
$$= \left[\frac{1}{3}y + \frac{y^5}{5}\right]_0^1$$
$$= \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

§ 5.4 Changing order

2b) Find $\displaystyle\int_{-3}^{1}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} x^2\,dx\,dy$

Sol.



NOT good to change order

$$\int_{-3}^{1}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} x^2\,dx\,dy$$

$$= \int_{-3}^{1} \left[\frac{x^3}{3}\right]_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} dy$$

$$= \frac{2}{3}\int_{-3}^{1} (9-y^2)^{3/2}\,dy$$

Put $y = 3\sin\theta$, $(9-y^2)^{3/2} = (9-9\sin^2\theta)^{3/2} = (9\cos^2\theta)^{3/2}$

$= 27\cos^3\theta$

$$= \frac{2}{3}\int_{-\pi}^{\sin^{-1}\frac{1}{3}} 27\cos^3\theta \cdot 3\cos\theta\,d\theta$$

$$= 54\int_{-\pi}^{\sin^{-1}\frac{1}{3}} \cos^4\theta\,d\theta$$

$$= 54\int_{-\pi}^{\sin^{-1}\frac{1}{3}} \frac{\cos4\theta+1}{8} + \cos2\theta + \frac{1}{4}\,d\theta$$

$\cos^4\theta$

$= (\cos^2\theta)^2$

$= \left(\frac{\cos2\theta+1}{2}\right)^2$

$= \frac{\cos^2 2\theta}{4} + \cos2\theta + \frac{1}{4}$

$= \frac{\cos4\theta+1}{8} + \cos2\theta + \frac{1}{4}$

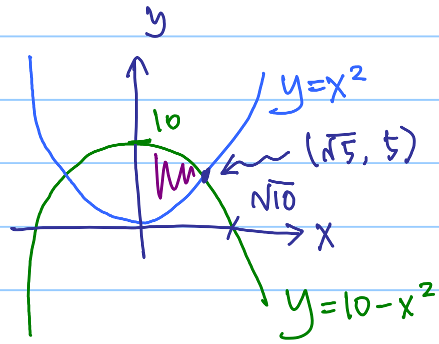
$$= 54\left[\frac{\sin4\theta}{32} + \frac{\theta}{8} + \frac{\sin2\theta}{2} + \frac{\theta}{4}\right]_{-\pi}^{\sin^{-1}\frac{1}{3}}$$

$= 33.607625$ (By calculator)

Q8 Compute $\iint_D f(x,y)\,dA$, $\quad f(x,y) = y^2\sqrt{x}$

$D = \{(x,y) \mid x>0,\ y>x^2,\ y<10-x^2\}$

Sol:



$$\iint_D f(x,y)\,dA$$

$$= \int_0^{\sqrt{5}} \int_{x^2}^{10-x^2} y^2\sqrt{x}\,dy\,dx$$

$$= \int_0^{\sqrt{5}} \left[\frac{y^3\sqrt{x}}{3}\right]_{x^2}^{10-x^2} dx$$

$$= \int_0^{\sqrt{5}} \frac{(10-x^2)^3\sqrt{x} - x^6\sqrt{x}}{3}\,dx$$

$$= 308.207$$

§5.5 Triple integral

Q4 $\quad \iiint_B z e^{x+y}\,dx\,dy\,dz \qquad B = [0,1]\times[0,1]\times[0,1]$

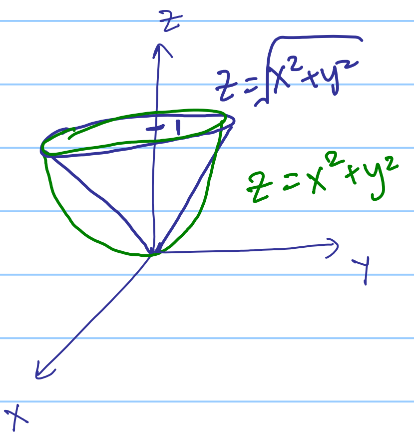
$$= \int_0^1\int_0^1\int_0^1 z e^{x+y}\,dx\,dy\,dz$$

$$= \left(\int_0^1 z\,dz\right)\left(\int_0^1 e^x\,dx\right)\left(\int_0^1 e^y\,dy\right)$$

$$= \left[\frac{z^2}{2}\right]_0^1 \cdot \left[e^x\right]_0^1 \cdot \left[e^y\right]_0^1$$

$$= \frac{1}{2}\cdot(e-1)\cdot(e-1) = \frac{(e-1)^2}{2}$$

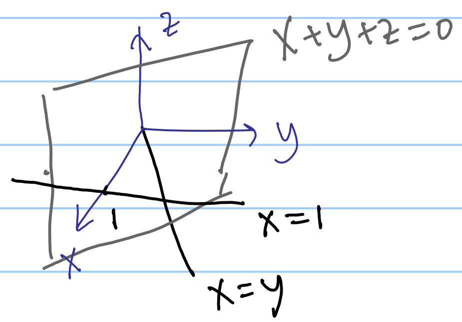
Q5 Describe: region between $z = \sqrt{x^2+y^2}$ & $z = x^2+y^2$



$$1 \le y \le -1$$
$$-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}$$
$$x^2+y^2 \le z \le \sqrt{x^2+y^2}$$

Q11 Find the volume of the solid bounded by
$x=y$, $z=0$, $y=0$, $x=1$ & $x+y+z=0$

Sol:



$$\int_0^1\int_0^x\int_0^{-x-y} 1 \, dz\,dy\,dx$$

$$= \int_0^1\int_0^x -x-y \, dy\,dx$$

$$= \int_0^1 \left[-xy - y^2/2\right]_0^x \, dx$$

$$= \int_0^1 -x^2 - \frac{x^2}{2} \, dx = \int_0^1 -\frac{3}{2}x^2 \, dx$$

$$= -\left[\frac{1}{2}x^3\right]_0^1 = \frac{1}{2}$$

So the volume is $\frac{1}{2}$