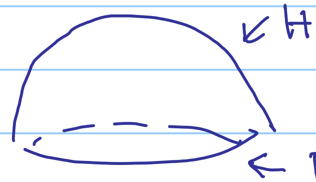
MATH20E §7.6

Q7) The surface S is



let H be the upper hemisphere, D be the base

(H) Note in P.496 1(d)

$$d\vec{S} = \vec{n}\, dS$$

where $\vec{n}$ is unit normal of S

Since H is hemisphere of unit ball

$$\therefore \vec{n} = (x, y, z)$$

$$\text{So } \vec{F} \cdot d\vec{S} = \vec{F} \cdot \vec{n}\, dS$$
$$= (x+3y^5, y+10xz, z-xy) \cdot (x, y, z)\, dS$$
$$= (x^2 + 3xy^5 + y^2 + 10xyz + z^2 - xyz)\, dS$$

Since $x^2+y^2+z^2=1$ on H

$$= (1+3xy^5 - 9xyz)\, dS$$

$$\therefore \iint_H \vec{F} \cdot d\vec{S} = \iint_H (1+3xy^5 - 9xyz)\, dS$$

Now we can consider the

$$\Phi(\theta, \phi) = (\cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi)$$

By P.497 3(a) $\quad dS = R^2 \sin\phi\, d\phi\, d\theta$

$$\therefore \iint_H \vec{F} \cdot d\vec{S} = \int_0^{2\pi} \int_0^{\pi/2} (1 + 3\cos\theta \sin^6\phi \sin^5\theta + 9\cos\theta \sin^2\phi \sin\theta \cos\phi) \cdot \sin\phi\, d\phi\, d\theta$$

we have $R=1$

The remaining steps would be the same.