MATH 20E MIDTERM I WINTER 2013 **NAME:** Mandy **SECTION:**

NO BOOKS, NO NOTES!! (except for one hand-written 'cheat sheet').
Justify your answers! Show your work!

10 1. (a) Find an equation of the plane tangent to the hyperboloid $x^2 + y^2 - 2z^2 = 18$ at $(2, 4, -1)$.
(b) Where does the tangent plane intersect the x-axis?

a) Let $f(x) = x^2 + y^2 - 2z^2$

$\nabla f(x) = (2x, 2y, -4z)$ 2

$\nabla f(2,4,-1) = (4, 8, 4)$ 2

tangent plane: $0 = (4,8,4) \cdot (x-2, y-4, z+1)$

$\Rightarrow x + 2y + z = 9$ 2

b) x-axis $\Leftrightarrow$ $y = z = 0$ 2

Sub $y=z=0$ into tangent plane eqn

$\Rightarrow x = 9$ 2

10 2. Let $\mathbf{G}(x, y, z) = xy^2\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$.
(a) Compute div $\mathbf{G} = \nabla \cdot \mathbf{G}$.
(b) Compute **curl** $\mathbf{G} = \nabla \times \mathbf{G}$.
(c) Is the vector field $\mathbf{G}$ a gradient field? Why, or why not?

a) $\nabla \cdot G = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (xy^2, xz, xy) = y^2$ 3

b) $\nabla \times G = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy^2 & xz & xy \end{vmatrix}$ 1

$= (0, -y, z - 2xy)$ 3

c) No 1

Since if G is a gradient field, $\nabla \times G = 0$

But from (b), we know that it isn't. 2

10

3. Let $f : \mathbf{R}^2 \to \mathbf{R}$ with $\nabla f(1,2) = (4,3)$ and $f(1,2) = 2$.
(a) Calculate the directional derivative of f at (1,2) in direction of the vector $(-1/\sqrt{2}, 1/\sqrt{2})$.
(b) Let $\mathbf{c}$ be the level curve of f through $(1,2)$, i.e. $f(\mathbf{c}(t)) = 2$ for all t and $\mathbf{c}(0) = (1,2)$. Find a vector which is parallel to $\mathbf{c}'(0)$. *Hint* : Calculate $\frac{d}{dt}(f(c(t))$ via two different methods.

a) directional derivative

$$= \nabla f(1,2) \cdot \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \qquad 2$$

$$= (4,3) \cdot \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} \qquad 2$$

b) $f(c(t)) = 0$

$$\Rightarrow \nabla f(c(0)) \cdot c'(0) = 0 \qquad 2$$

$$(4,3) \cdot c'(0) = 0 \qquad 2$$

So a vector parallel to $c'(0)$ is $(-3, 4)$ 2

or any scalar multiple of this vector.

4. Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be the function given by $f(x,y) = (xe^y, y\sqrt{x})$.
(a) Calculate its derivative $Df(1,0)$.
(b) Find a good approximation of $f(1.01, .02)$.

a) $$Df = \begin{pmatrix} e^y & xe^y \\ \frac{1}{2}\frac{y}{x} & \sqrt{x} \end{pmatrix}$$

4 for each term
1 for putting in correct position

$$Df(1,0) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad 1$$

b) $$f(1.01, 0.02) \approx f(1,2) + Df(1,0) \cdot \begin{pmatrix} 1.01 - 1 \\ 0.02 - 0 \end{pmatrix} \qquad 2$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0.01 \\ 0.02 \end{pmatrix} \qquad 1$$

$$= \begin{pmatrix} 1.03 \\ 0.02 \end{pmatrix} \qquad 1$$