

## HW3

### Section 2.3 question 4b

We must show that the partials are continuous in the domain:

$$\frac{\partial f}{\partial x} = \frac{1}{y} - \frac{y}{x^2} = \frac{x^2 - y^2}{x^2 y} \text{ which is continuous for } x \neq 0 \text{ and } y \neq 0;$$

$$\frac{\partial f}{\partial y} = -\frac{x}{y^2} + \frac{1}{x} = \frac{y^2 - x^2}{xy}, \text{ which is continuous for } x \neq 0.$$

Thus  $f(x)$  is  $C^1$  since its partial derivatives are continuous.

### Section 2.3 question 7b

The first row contains the partial derivatives of  $xe^y + \cos y$ . The second row contains those of  $x$ , the third row contains those of  $x + e^y$ . The first column contains the partial derivatives with respect to  $x$ , and the second column contains those with respect to  $y$ . Thus, the matrix of partial derivatives is

$$\begin{pmatrix} e^y & xe^y - \sin y \\ 1 & 0 \\ 1 & e^y \end{pmatrix}$$

### Section 2.3, question 20

We want to find  $\mathbf{T}$  in equation (4) in page 134. By linearity

$f(x) - f(x_0) = f(x - x_0)$ . Denoting  $x - x_0$  by  $h$ , we want to find  $\mathbf{T}$  so that

$$\lim_{h \rightarrow 0} \frac{|f(h) - \mathbf{T}h|}{\|h\|} = 0.$$

If we choose  $\mathbf{T} = f$ , the numerator vanishes for all  $h$ , so this  $\mathbf{T}$  satisfies the condition; that is, the derivative of a linear map is the map itself.

### Section 2.5, question 8

$$\frac{\partial u}{\partial \rho} = \frac{\partial u}{\partial x} \cos \theta \sin \phi + \frac{\partial u}{\partial y} \sin \theta \sin \phi + \frac{\partial u}{\partial z} \cos \phi$$

$$\frac{\partial u}{\partial \theta} = \sin \theta \sin \phi \rho \frac{\partial u}{\partial x} + \cos \theta \sin \phi \rho \frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial \phi} = \rho \cos \theta \cos \phi \frac{\partial u}{\partial x} + \rho \sin \theta \cos \phi \frac{\partial u}{\partial y} - \rho \sin \phi \frac{\partial u}{\partial z}$$

### Section 2.5, question 9

Substitute  $u = e^{x-y}$  and  $v = x - y$  to get

$$f \circ g = (\tan(e^{x-y} - 1) - e^{x-y}, (e^{x-y})^2 - (x - y)^2).$$

First, we have

$$Df(u, v) = \begin{pmatrix} \sec^2(u - 1) & -e^v \\ 2u & -2v \end{pmatrix}.$$

When  $(x, y) = (1, 1)$ , we have  $g(1, 1) = (1, 0)$ . Hence,

$$Df(1, 0) = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}.$$

Next, we calculate

$$Dg(x, y) = \begin{pmatrix} e^{x-y} & -e^{x-y} \\ 1 & -1 \end{pmatrix}$$

$$\text{So } Dg(1, 1) = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}.$$

Therefore,

$$D(f \circ g)(1, 1) = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2 & -2 \end{pmatrix}.$$

### Section 2.6, question 5b

We have  $z = (\cos x)(\cos y)$ , so  $z_x = -\sin x \cos y$  and, by symmetry,  $z_y = -\sin y \cos x$ . At  $(0, \frac{\pi}{2}, 0)$ ,  $z_x = 0$  and  $z_y = -1$ .

Therefore, the equation of the tangent plane is

$$z + y = \frac{\pi}{2}$$

### Section 2.6, question 6c

$$\text{We have } f_x = -\frac{2x}{(x^2 + y^2 + z^2)^2} = -\frac{2x}{r^4}, f_y = -\frac{2y}{r^4}, f_z = -\frac{2z}{r^4}.$$

Then  $\nabla f = -\frac{2}{r^4} (xi + yj + zk)$ .

**Section 2.6, question 20**

The direction in which the altitude is increasing most rapidly at the point  $(x,y)$  is

$$\nabla z(x,y) = (-2ax, -2by).$$

At that point  $(1,1)$ ,  $\nabla z(1,1) = -2(a,b)$ , so the desired direction is

$-\frac{ai+bj}{\sqrt{a^2+b^2}}$ . If a marble were released at  $(1,1)$ , it will roll in the direction at which the altitude is decreasing most rapidly, so the marble will roll down in the direction  $-\nabla z(1,1)$ , i.e.  $\frac{ai+bj}{\sqrt{a^2+b^2}}$ .