

# MATH 115 Problem Set 9

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1. Find the Fourier series of the following functions:

$$f(t) = t^2, \quad 0 < t < 2, \quad f(t+2) = f(t).$$

2. The output from an electronic oscillator takes the form of a sine wave  $f(t) = \sin t$  for  $0 < t \leq \frac{\pi}{2}$ , it then drops to zero and starts again. Find the **complex** Fourier series of this wave form.
3. Use the method of jumps to find the half-range cosine series of the function  $g(t) = \sin t$  defined in the interval of  $0 < t < \pi$ .
4. Use the Fourier series of  $f(t) = t$  for  $-1 < t < 1$ , and  $f(t+2) = f(t)$  to show that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}.$$

5. Find the Fourier series for  $f(t) = |t|$  for  $-\pi < t < \pi$ , and  $f(t+2\pi) = f(t)$ . Does the series resulting from a term-by-term differentiation (of the series of  $f(t)$ ) converge to  $f'(t)$ ?
6. Find the steady-state solution of

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = f(t),$$

where  $f(t) = t$ ,  $-\pi \leq t < \pi$ ,  $f(t+2\pi) = f(t)$ .

7. Use the Fourier series method to solve the following boundary value problem

$$\frac{d^4y}{dx^4} = \frac{P}{EIL}x,$$

$$y(0) = 0, y(L) = 0; \quad y''(0) = 0, y''(L) = 0.$$