

MATH 115 Problem Set 8

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1. Find the following inverse Laplace transforms:

(a) $\mathfrak{L}^{-1} \left[\frac{s^2}{(s^2+a^2)^2} \right]$.

(b) $\mathfrak{L}^{-1} \left[\frac{1}{s^2(s^2+1)} \right]$.

2. Use the complex inversion integral (residue calculus), find the inverse of the following Laplace transform:

$$\frac{1}{(s^2 + 9)(s^2 + 4)}.$$

3. Use the Laplace transform to solve the following differential equations:

$$y'''' - y = \delta(t - t_0), \quad y(0) = y'(0) = y''(0) = y'''(0) = 0, \quad \text{where } \delta \text{ is the delta function.}$$

4. Use the Laplace transform to solve the following system of differential equations:

$$\begin{aligned} \frac{dy}{dt} &= 2y - 3z, \\ \frac{dz}{dt} &= -2y + z, \\ y(0) &= 8, z(0) = 3. \end{aligned}$$

5. The damped harmonic oscillator is governed by

$$mx'' + bx' + kx = f(t); \quad \text{with } x(0) = x'(0) = 0.$$

Find the solution $x(t)$ when $b = 0$ and $f(t) = F_0 u(t - t_0)$ where $u(t - t_0)$ is the step function and F_0 is some constant.

6. Find the solution of the integrodifferential equation

$$y'(t) - \int_0^t y(\tau) \cos(t - \tau) d\tau = 0, \quad y(0) = 1.$$

7. Find the Fourier series of the follow function:

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 2 & 0 < x < \pi \end{cases}, \quad f(x + 2\pi) = f(x).$$