

# MATH 115 Problem Set 6

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1. Find the specific solutions of the following nonhomogeneous differential equations:

$$y'' - 4y = 8e^{2x}, \quad y(0) = 4, \quad y'(0) = 6.$$

2. Solve the following differential equations with the method of variation of parameters:

$$y'' + y = \sec x.$$

3. Solve

$$\begin{aligned} y'(x) + z'(x) + y(x) + z(x) &= 1, \\ y'(x) - y(x) - 2z(x) &= 0, \\ y(0) = 1, \quad z(0) &= 0. \end{aligned}$$

4. Solve

$$\begin{aligned} y'(x) - z'(x) - 2y(x) + 2z(x) &= 1 - 2x, \\ y''(x) + 2z'(x) + y(x) &= 0, \\ y(0) = z(0) = y'(0) &= 0. \end{aligned}$$

5. Consider

$$\frac{d^4 y}{dx^4} = -4a^4 y,$$

where  $a$  is a positive constant. Find the general solution of this equation.

6. Solve

$$y''(t) + y(t) = \begin{cases} 1 - \frac{t^2}{\pi^2} & \text{if } 0 \leq t \leq \pi, \\ 0 & \text{if } t > \pi, \end{cases}$$
$$y(0) = y'(0) = 0.$$

This may be interpreted as an undamped system on which a force acts during some interval of time, for instance, the force acting on a gun barrel when a shell is fired, the barrel being braked by heavy springs (and then damped by a dashpot which we disregard for simplicity).

Note that  $y$  and  $y'$  are continuous at  $t = \pi$ .