MATH 115 Problem Set 4

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1. Find the residue of
$$f(z) = \frac{z+1}{z^2(z-2)}$$
(a) at $z = 0$ and (b) at $z = 2$.

2. Use the theory of residue to evaluate the following definite integrals
   (a) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$,
   (b) $\int_0^{\pi} \frac{\cos 2\theta d\theta}{1-2a\cos\theta+a^2}$, where $-1 < a < 1$,
   (c) $\int_0^{\pi} \sin^{2n}\theta d\theta$, where $n = 1, 2, \ldots$.

3. Show that
   (a)
$$\int_{-\infty}^{\infty} \frac{x^2+1}{x^4+1} dx = \sqrt{2}\pi;$$
   (b)
$$\int_0^{\infty} \frac{ab}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{2(a+b)} \text{ for } a, b \in \mathbb{R}$$

4. Evaluate
   (a)
$$\int_{-\infty}^{\infty} \frac{e^{3ix}}{x-2i} dx;$$
   (b)
$$\int_{-\infty}^{\infty} \frac{\cos mx}{(x^2+a^2)(x^2+b^2)} dx \text{ for } a, b \in \mathbb{R}.$$

5. Use a rectangular contour to show that
$$\int_{-\infty}^{\infty} \frac{\cos mx}{e^{-x}+e^{x}} dx = \frac{\pi}{e^{m\pi/2}+e^{-m\pi/2}}.$$

6. Use a pie-shaped contour with $\theta = \frac{2\pi}{3}$ to show that
$$\int_0^{\infty} \frac{1}{x^3+1} dx = \frac{2\sqrt{3}\pi}{9}.$$