

MATH 115 Problem Set 3

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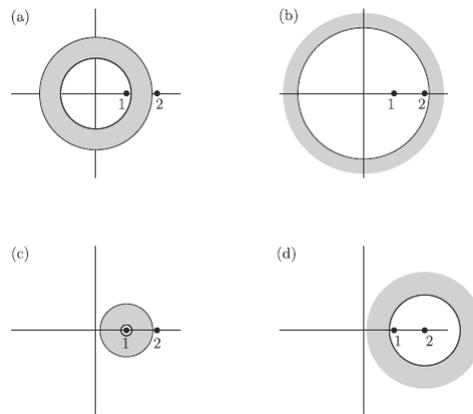
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1. Expand $f(z) = \frac{z-1}{z+1}$ in a Taylor's series around $z = 1$. Determine the radius of convergence of the series.
2. Consider $\log z : \mathbb{C} \rightarrow \mathbb{R}_{\leq 0}$ with the choice $\log 1 = 0$ where $\mathbb{R}_{\leq 0} = \{x \in \mathbb{R} | x \leq 0\}$. Show that its power series expansion at $z = 1$ is

$$\log(z) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (z-1)^n.$$

Where is this expansion valid? What happens if one chooses the branch $\log 1 = 2\pi i$?

3. Without obtaining the series, determine the radius of convergence of each of the following expansions:
 - (a) $\tan^{-1} z$ around $z = 1$;
 - (b) $\frac{1}{e^z-1}$ around $z = 4i$;
 - (c) $\frac{z}{z^2+2z+10}$ around $z = 0$.
4. Expand $f(z) = \frac{1}{z^2-3z+2}$ in the following regions.



5. Expand

$$f(z) = \frac{1}{z^2(z-i)}$$

in two different Laurent expansions around $z = i$ and tell where each converges.

6. What is the residue of

$$f(z) = \frac{1}{z - \sin z}$$

at $z = 0$?

7. Use the residue to evaluate $\oint_C f(z)dz$ if C is the circle $|z| = 4$ for each of the following functions:

(a) $\frac{z}{z^2-1}$,

(b) $\frac{1}{z(z-2)^3}$

8. Show that

$$\oint_C \frac{1}{(z^{100} + 1)(z - 4)} dz = \frac{-2\pi i}{4^{100} + 1},$$

if C is the circle $|z| = 3$.