MATH 115 Problem Set 11

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1. Let $u(x,y)$ e the steady-state temperature in a thin plate in the shape of a semi-infinite strip. Let the surface heat transfer takes place at the face so that

$$\frac{\partial u(x,y)}{\partial x^2} + \frac{\partial u(x,y)}{\partial y^2} - bu(x,y) = 0, \quad 0 \leq x \leq 1; y \geq 0.$$

If u is bounded as $y \to \infty$ and satisfies the conditions

$$u(0,y) = 0, \quad u_x(1,y) = -hu(1,y), \quad u(x,0) = 1.$$

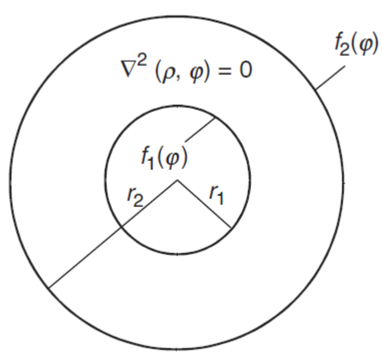
Show that

$$u(x,y) = 2h \sum_{n=1}^{\infty} \frac{A_n}{\alpha_n} e^{-\sqrt{b+\alpha_n^2}y} \sin \alpha_n x,$$

where $A_n = \frac{1-\cos\alpha_n}{h+\cos^2\alpha_n}$ and $\alpha_1, \alpha_2, \alpha_3$ are positive roots of the equation $\tan\alpha = -\frac{\alpha}{h}$.

2. Find the steady-state distribution of temperature in a sector of a circular plate of radius 10 and angle $\frac{\pi}{4}$ if the temperature is maintained at $0°$ along the radii and at $100°$ along the curved edge.

3. Suppose that the temperatures along the inner circle of radus of r_1 and along the outer circle of radius r_2 of an annulus are maintained at $f_1(\phi)$ and $f_2(\phi)$, respectively, as shown in the figure below. Determine the steady-state temperature in the annulus, if $f_1(\phi) = 0$, $f_2(\phi) = \sin\phi$, $r_1 = 1$, $r_2 = 2$.



4. If the membrane and its frame are moving as a rigid body with unit velocity perpendicular to the membrance and the frame is suddenly brough to rest, then the membrane will start to vibrate. The vibration can be modeled with the following boundary value problem:

$$\begin{aligned} D.E.: &\quad \nabla^2 z(\rho,t) = \frac{1}{a^2}\frac{\partial^2}{\partial t^2} z(\rho,t), \\ B.C.: &\quad z(c,t) = 0, \\ I.C.: &\quad z(\rho,0) = 0, \quad \frac{\partial z}{\partial t}|_{t=0} = 1. \end{aligned}$$

Find the displacements $z(\rho,t)$.

5. A particular of mass m is contained in a right circular cylindrical box of radius R and height H. The particle is described by a wavefunction satisfying the Schrödinger wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r,\theta,z) = E\psi(r,\theta,z)$$

and the condition that the savefunction go to zero over the surface of the box. Find the lowest permitted energy.