MATH 115 Problem Set 1

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1. Find all the distinct fourth roots of $8 - i8\sqrt{3}$.

2. Show that

$$\sum_{k=0}^{n} \cos k\theta = \frac{1}{2} + \frac{\sin[(n + \frac{1}{2})\theta]}{2\sin\frac{1}{2}\theta}$$

$$\sum_{k=0}^{n} \sin k\theta = \frac{1}{2}\cot\frac{1}{2}\theta - \frac{\cos[(n + \frac{1}{2})\theta]}{2\sin\frac{1}{2}\theta}$$

3. a. Express the function $f(x, y) = (x^2 - y^2) + 2ixy$ in therms of $(z, \bar{z})$.
   b. Express the function $f(z, \bar{z}) = z\bar{z}$ in terms of (x, y).

4. With $z = x + iy$, verify $\cos z = \cos x \cosh y - i \sin x \sinh y$.

5. Find the image of horizontal lines $y = 0, 1, 2, \ldots$ and vertical lines $x = 0, 1, 2, \ldots$ under

$$f : \mathbb{C} \to \mathbb{C} \qquad z \mapsto z^2.$$

6. Show that the real and the imaginary parts of $\frac{1}{z+2}$ satisfy the Cauchy-Riemann conditions.

7. In which quadrants of the complex plane is $f(z) = |x| - i|y|$ a holomorphic function?

8. If $f(z) = u(x, y) + iv(x, y)$ is analytic, show that both u and v satisfy the Laplace equation.