§4.1

4) $f(x) = (x^3-8)^4$

$f'(x) = 4(x^3-8)^3(3x^2)$

$f'(x) = 0$

$\Rightarrow x^3 - 8 = 0$ or $x^2 = 0$

$\therefore x = 2$ or $x = 0$

$\therefore$ critical pt is $x = 0, 2$

$$\begin{array}{ccccc} & - & & - & & + \\ \hline & & 0 & & 2 & \end{array}$$

$\therefore f(2)$ is local min

$f(0)$ is not local extremum

11 b) $f(x) = e^{-x^2}$

$f'(x) = -2xe^{-x^2} = 0$

$\Rightarrow x = 0$

$\therefore$ critical pt is $x = 0$

$f''(x) = -2e^{-x^2} + 4x^2e^{-x^2} = e^{-x^2}(-2+4x^2) = 0$

$\therefore -2 + 4x^2 = 0$

$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$

$\therefore$ pt. of inflection: $x = \pm \frac{1}{\sqrt{2}}$

13) $f(x) = x^4 + x^3 - 3x^2 + 2$

$f'(x) = 4x^3 + 3x^2 - 6x$

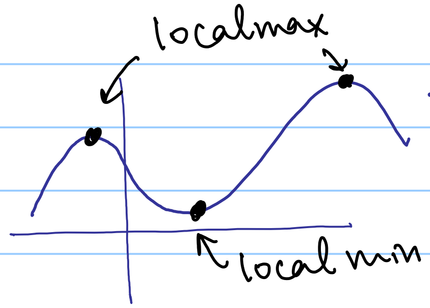
$f''(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x-1)(x+1)$

$\therefore x = -1, \frac{1}{2}$

$$f'' \quad \begin{array}{ccccc} & + & & - & & + \\ \hline & & -1 & & \frac{1}{2} & \end{array}$$

$\therefore x = -1, \frac{1}{2}$ are inflection pts

18)



23) a)

$f(x) = x - b\ln x$

$f'(x) = 1 - \frac{b}{x} = 0$

$\Rightarrow x = b$

$\therefore$ critical pt. $x = b$

b)

$f''(x) = \frac{b}{x^2} > 0$ for all $x > 0$ $(\because b > 0)$

$\therefore$ local min at $x = b$

41)

$f(x) = x^2 + ax + b$

$f'(x) = 2x + a = 0$

$x = -\frac{a}{2} = 6 \Rightarrow a = -12$

$f(6) = 6^2 - 12(6) + b = 36 - 72 + b = -5$

$\Rightarrow b = 31$

$\therefore f(x) = x^2 - 12x + 31$

47)

$f = B$

$f' = A$

$f'' = C$

49)

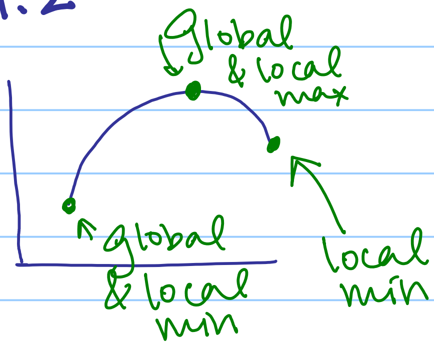
$f = II$

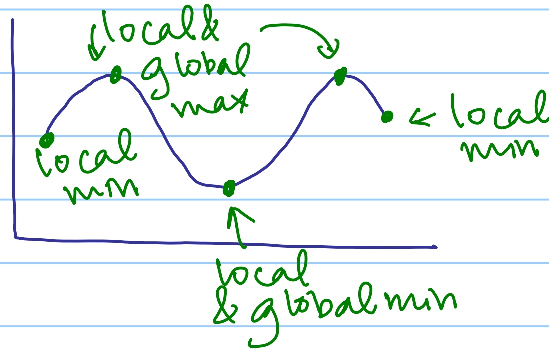
$f' = III$

$f'' = I$

§4.2

1)





19) $f(x) = x - \ln x$ for $x > 0$

$f'(x) = 1 - \frac{1}{x} = 0$

$\Rightarrow x = 1$

since $x > 0$

$\therefore$ f has local min at $x = 1$

$\lim_{x \to 0} (x - \ln x) \to \infty$

$\lim_{x \to +\infty} (x - \ln x) \to \infty$

$\therefore$ f has global min at $x=1$ with $f(1) = 1$

no global max

25) $y' = -32t + 50 = 0$

$\Rightarrow t = \frac{50}{32} = \frac{25}{16}$

$\therefore$ It will go $y\left(\frac{25}{16}\right) = -16\left(\frac{50}{32}\right)^2 + 50\left(\frac{50}{32}\right) + 5 \approx 44.1$ feet

§4.4

11) $y = x^3 - 4x^2 + 4x$

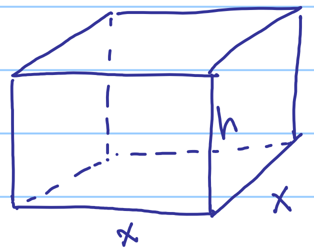
$y' = 3x^2 - 8x + 4$

$= (3x-2)(x-2) = 0$

$\Rightarrow \quad x = \frac{2}{3}$ or 2 critical pt.

$y(0) = 0, \quad y(\frac{2}{3}) = \frac{32}{27}, \quad y(2) = 0, \; y(4) = 16$

$\therefore \quad 0 \le y \le 16$

21) 

Surface area $= x^2 + 4xh$

Volume $= x^2 h$

Volume $= 8\,cm^3 \Rightarrow \quad h = 8/x^2$

$\therefore$ Substituting into surface area

$\Rightarrow S(x) = x^2 + 4x(8/x^2) = x^2 + \frac{32}{x}$

$S'(x) = 2x - \frac{32}{x^2} = 0$

$2x^3 - 32 = 0$

$2(x^3 - 16) = 0$

$\Rightarrow \quad x = \sqrt[3]{16}$ cm & $h = 8/x^2 = \frac{\sqrt[3]{16}}{2}$ cm

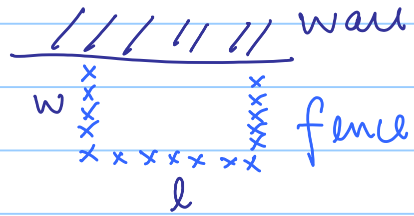
Check by 2nd derivative test.

$S''(x) = 2 + \frac{64}{x^3}$

$S''(\sqrt[3]{16}) > 0 \quad (\because$ all terms $> 0)$

$\therefore \quad x = \sqrt[3]{16}$ gives a min value of S.

32)



let w and l be width & length, respectively

$$\text{length of fence} = 2w + l = 100 \text{ feet}$$
$$l = 100 - 2w$$

$$A = \text{Area} = wl$$
$$= w(100-2w) = 100w - 2w^2$$
$$\frac{dA}{dw} = 100 - 4w = 0$$
$$\Rightarrow w = 25, \; l = 50$$

Second derivative test : $\frac{d^2A}{dw^2} = -4 < 0 \Rightarrow$ local max

Since the graph is parabolic,

So local max is global max

$\therefore$ Area is max when $w=25, l=50$ with area $= 25 \cdot 50 = 1250 \text{ ft}^2$

37) let the pt. be (x, x^2) on the parabola $y = x^2$

$$\text{distance} = s(x) = \sqrt{(x-3)^2 + (x^2-0)^2} = \sqrt{(x-3)^2 + x^4}$$

To find max/min pt. of distance,

It is the same as finding max/min of $(\text{distance})^2$

$\therefore$ Consider $Q(x) = (x-3)^2 + x^4$

$$Q'(x) = 2(x-3) + 4x^3 = 0$$
$$\Rightarrow x = 1$$
$$Q''(x) = 2 + 12x^2 > 0 \Rightarrow \text{local min}$$

$\therefore$ The pt. $(1,1)$ has closest distance at $y = x^2$ to the pt. $(3, 0)$.