Prictice Final for MATH 10A

1) a) $\lim_{x\to 0} \frac{x^2+3x}{2x^2-1} = \frac{0}{-1} = 0$

b) $\lim_{x\to\infty} \sin(x)$ does not exist since the sine curve still jumping between 1 & -1 when x is large.

c) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{3+3} = \frac{1}{6}$

2) a) $p'(x) = \frac{1}{f(x)} \cdot f'(x)$

$p'(3) = \frac{1}{f(3)} \cdot f'(3)$

$= \frac{1}{2}(-5) = -\frac{5}{2}$

b) $g'(x) = \frac{f'(x)\cdot x - f(x)}{x^2}$

$g'(3) = \frac{f'(3)\cdot 3 - f(3)}{3^2} = \frac{(-5)\cdot 3 - 2}{3^2} = -\frac{17}{9}$

3) a) $f'(1) > f'(2)$

b) $\frac{f(1)-f(0)}{1-0} > \frac{f(2)-f(1)}{2-1}$

c) $f'(2) < \frac{f(2)-f(1)}{2-1}$

4) a) First: $\lim_{x\to 0^-} f(x) = 0$ & $\lim_{x\to 0^+} f(x) = 0$

So f is continuous at $x=0$ for all a

And left derivative $= \lim_{h\to 0^-} \dfrac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0^+} \dfrac{ah}{h} = \lim_{h\to 0^+} a = a$

right derivative $= \lim_{h\to 0^+} \dfrac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0^-} \dfrac{h^2+4h}{h} = 4$

$\therefore f$ is differentiable at 0 when $a=4$

b) First, $\lim_{x\to -2^-} f(x) = b$

$\lim_{x\to -2^+} f(x) = (-2)^2+4(-2) = 4-8 = -4$

$\therefore f$ is continuous at -2 when $b=-4$

And left hand derivative $= \lim_{h\to 0^-} \dfrac{f(-2+h)-f(-2)}{h} = 0$

right hand derivative $= \lim_{h\to 0^+} \frac{f(-2+h)-f(-2)}{h}$

$= \lim_{h\to 0^+} \frac{(-2+h)^2-4(-2+h)-[(-2)^2-4(-2)]}{h}$

$= \lim_{h\to 0^+} \frac{-4h+h^2-4h}{h}$

$= \lim_{h\to 0^+} \frac{h^2}{h} = 0$

So f is diff at $x=-2$ when $b=-4$

5 (a) $y = e^{x/3}$

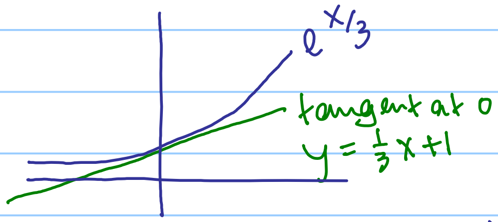
$y' = \frac{1}{3}e^{x/3}$

$y'(0) = \frac{1}{3}$

$\therefore$ tangent line eqn : $y = y'(0)(x-0)+y(0)$

$= \frac{1}{3}x+1$

b)



For $y=e^{x/3}$

$y' = \frac{1}{3}e^{x/3}$

$y'' = \frac{1}{9}e^{x/3} > 0$ for all x

so the graph of $e^{x/3}$ is concave up.

$\therefore$ always have graph above the tangent

$\therefore$ approx. value by tangent at $x=1$ is $\frac{1}{3}(1)=1 = \frac{4}{3}$

$\therefore \quad e^{1/3} > \frac{4}{3}$

6) $\cos(xy) = y^2$

Diff. both side with respect to x, get

$$-\sin(xy)\cdot\left(y + x\frac{dy}{dx}\right) = 2y\cdot\frac{dy}{dx}$$

$$-y\sin(xy) = 2y\frac{dy}{dx} + x\sin(xy)\cdot\frac{dy}{dx}$$

$$= (2y + x\sin(xy))\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y\sin(xy)}{2y + x\sin(xy)}$$

7) $f(x) = x^2e^{-x}$

$$f'(x) = 2xe^{-x} - x^2e^{-x}$$

$$= e^{-x}(2x - x^2) = 0$$

$$\Rightarrow \quad 2x - x^2 = 0$$

$$x(2-x) = 0$$

$\therefore$ Critical pts are $x=0$ & $x=2$

By first derivative test

sign for f' $-$ (0) $+$ (2) $-$

So $x=0$ is a local min

$x=2$ is a local max

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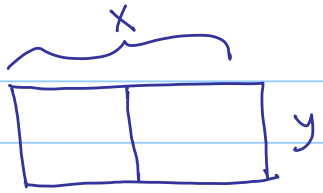

length of Fence $= 2x+3y = 1200$

$x = 600 - \frac{3}{2}y$

Area $= xy = (600 - \frac{3}{2}y)\,y$

$A(y) = (600 - \frac{3}{2}y)\,y = 600y - \frac{3}{2}y^2$

$A'(y) = 600 - 3y = 0$

$\Rightarrow y = 200$

$A''(y) = -3 < 0$

By 2nd derivative test, $y = 200$ is a local max

$0 \le y \le 400$ because x, y are lengths

$\Rightarrow x, y \geq 0$

$x \geq 0 \Rightarrow 600 - \frac{3}{2}y \geq 0$

$\Rightarrow y \le 400$

& $A(0) = 0$, $A(400) = 0$

$\therefore$ Dimensions of the plot would be

$x = 600 - \frac{3}{2}(200) = 300$ feet, $y = 200$ feet

to have max area