Math 10a Midterm 2 Version A

1) a) $y' = (\ln 20)\,20^x + 20x^{19}$

b) $\left(\frac{1}{x^7} - \frac{5}{x^4}\right)6^x = \left(x^{-7} - 5x^{-4}\right)6^x$

So the derivative is:

$$\left(-7x^{-8} + 20x^{-5}\right)6^x + \ln 6\left(x^{-7} - 5x^{-4}\right)6^x$$

c) $y' = \left(e^{\sqrt{2x^6+12}}\right)\cdot\left(\frac{1}{2}(2x^6+12)^{-\frac{1}{2}}\right)\cdot(12x^5)$

$$= \frac{6x^5 e^{\sqrt{2x^6+12}}}{\sqrt{2x^6+12}}$$

2) a) $g'(x) = \dfrac{8x(x+2) - 4x^2}{(x+2)^2}$

when $x=1$, $g'(1) = \dfrac{8\cdot(3) - 4}{3^2} = \dfrac{20}{9}$

and when $x=1$, $g(1) = \dfrac{4}{1+2} = \dfrac{4}{3}$

$\therefore$ Eqn for tangent: $\dfrac{y - \frac{4}{3}}{x-1} = \dfrac{20}{9}$

$$y = \frac{20}{9}(x-1) + \frac{4}{3}$$

$$= \frac{20}{9}x - \frac{8}{9}$$

b) tangent line is horizontal

$\Rightarrow g'(x) = 0$

i.e. $\dfrac{8x(x+2) - 4x^2}{(x+2)^2} = 0$

$$8x(x+2) - 4x^2 = 0$$

$$4x^2 + 16x = 0$$

$$4x(x+4) = 0 \Rightarrow x = 0 \text{ or } x = -4$$

3) a) $-6 < x < -4,\ 2 < x < 4$

b) $-5 < x < -2,\ 0 < x < 1,\ 3 < x$

3) a) $P'(t)$ means the rate of change of population at time $t$ (year after 1950)
unit = people/year

b) $P'(20)$ is negative means the population is decreasing at the year 1970 (or at 20th year after 1950)

c) $P'(30) = 0$ means the population remains constant at 1980 (or at 30th year after 1950)