Math 10A Week 9

§3.9 Linear approximation

Suppose f is differentiable near a.
Then tangent line approximation is

$$f(x) \approx \underbrace{f(a) + f'(a)(x-a)}_{\text{local linearization of } f \text{ near } x=a}$$

Error: $E(x) = f(x) - f(a) + f'(a)(x-a)$

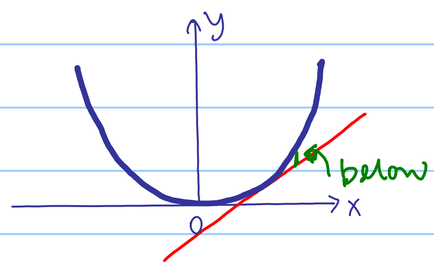
Q2 Find the tangent line approximation for $\sqrt{1+x}$ near $x=0$.

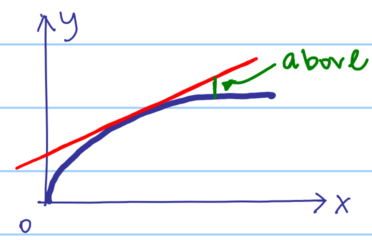
$$f(x) = \sqrt{1+x}$$
$$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} \Rightarrow \begin{cases} f(0) = 1 \\ f'(0) = \frac{1}{2} \end{cases}$$

$\therefore$ tangent line approx:

$$f(x) \approx f(0) + f'(0)(x-0)$$
$$\sqrt{1+x} \approx 1 + \frac{x}{2}$$

Q8 Local linearization gives value too small for x^2 and too large for $\sqrt{x}$.
Draw picture to explain why.





The grap of x^2 is concave up and lies above tange line, so the linearization is always small.
The grap of $\sqrt{x}$ is concave down lies below tange line, so the linearization is always large.

Q13 a) Find the tangent line approximation to $\cos x$ at $x = \frac{\pi}{4}$.

b) Use a graph to explain how you know whether the tangent line approx is an under- or over-estimate for $0 \le x \le \frac{\pi}{2}$

a) $f(x) = \cos x$ $\Rightarrow$ $f(\frac{\pi}{4}) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

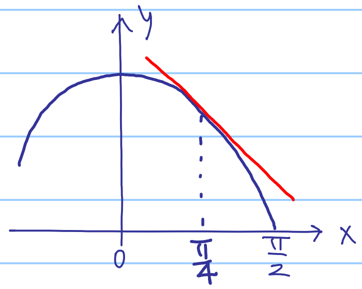
$f'(x) = -\sin x$ $\quad$ $f'(\frac{\pi}{4}) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\therefore$ tangent line approx:

$$f(x) \approx f(\tfrac{\pi}{4}) + f'(\tfrac{\pi}{4})(x - \tfrac{\pi}{4})$$

$$\cos x \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}(1 + \frac{\pi}{4})$$

b)



tangent is above the curve

$\Rightarrow$ overestimate