Math 10a Week 8

§3.5 The trigonometric functions

For x in radians, $\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x} = \sec^2 x$

Q3 $s(\theta) = \cos\theta \sin\theta$

$$s'(\theta) = \frac{d}{d\theta}(\cos\theta) \cdot \sin\theta + \cos\theta \frac{d}{d\theta}(\sin\theta)$$
$$= -\sin^2\theta + \cos^2\theta = \cos 2\theta$$

Q19 $f(x) = \tan(\sin x)$

$$f'(x) = \frac{1}{\cos^2(\sin x)} \cdot \frac{d}{dx} \cdot (\sin x)$$
$$= \frac{1}{\cos^2(\sin x)} \cdot \cos x$$

Q37 $f(x) = \sqrt{\frac{1-\sin x}{1-\cos x}}$

$$f'(x) = \frac{1}{2}\left(\frac{1-\sin x}{1-\cos x}\right)^{-\frac{1}{2}} \cdot \frac{d}{dx}\left(\frac{1-\sin x}{1-\cos x}\right)$$
$$= \frac{1}{2}\left(\frac{1-\sin x}{1-\cos x}\right)^{-\frac{1}{2}} \cdot \frac{-\cos x(1-\cos x) - \sin x(1-\sin x)}{(1-\cos x)^2}$$
$$= \frac{1}{2}\left(\frac{1-\sin x}{1-\cos x}\right)^{-\frac{1}{2}} \cdot \left(\frac{1-\cos x-\sin x}{(1-\cos x)^2}\right)$$

Q43 Find a function $F(x)$ satisfying $F'(x) = \sin(4x)$

Guess: $F(x) = \cos(4x)$

$F'(x) = -4\sin(4x)$

$\rightsquigarrow$ so $F(x) = -\frac{1}{4}\cos(4x)$

§3.6 Inverse function

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$$

Q9 $\qquad j(x) = \ln(e^{ax}+b)$ $\qquad$ a, b constant.

$$j'(x) = \frac{1}{e^{ax}+b} \cdot (ae^{ax}) = \frac{ae^{ax}}{ae^{ax}+b}$$

Q30 $\qquad f(x) = \ln(\ln t) + \ln(\ln 2)$

$$f'(x) = \frac{1}{\ln t} \cdot \frac{1}{t} + 0 = \frac{1}{t \ln t}$$

Q45a) For $x>0$, find and simply the derivative of
$f(x) = \arctan x + \arctan(\frac{1}{x})$

b) What does your result tell you about f ?

a) $$f'(x) = \frac{1}{1+x^2} + \frac{1}{1+(\frac{1}{x})^2} \cdot \left(\frac{-1}{x^2}\right)$$
$$= \frac{1}{1+x^2} - \frac{1}{x^2+1} = 0$$

b) f is a constant function

§3.7 Implicit function

Q11 find $\frac{dy}{dx}$

$$x\ln y + y^3 = \ln x$$

Diff. both side with respect to x,

$$\ln y + x\cdot\frac{1}{y}\frac{dy}{dx} + 3y^2\cdot\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{x}{y}+3y^2\right) = \frac{1}{x} - \ln y$$

$$\frac{dy}{dx} = \left(\frac{1}{x}-\ln y\right)\Big/\left(\frac{x}{y}+3y^2\right) = \frac{1-x\ln y}{x}\cdot\frac{y}{x+3y^3}$$

Q13 find $\frac{dy}{dx}$

$$\cos^2 y + \sin^2 y = y+2$$

Diff. both side with respect to x,

$$2\cos y\cdot(-\sin y)\frac{dy}{dx} + 2\sin y\cdot\cos y\cdot\frac{dy}{dx} = \frac{dy}{dx}$$

$$0 = \frac{dy}{dx}$$

Can do better

Because $\cos^2 y + \sin^2 y = 1$

$$\therefore \quad 1 = y+2$$

$$\Rightarrow \quad y=-1 \quad \Rightarrow \quad \frac{dy}{dx} = 0$$

Q25 Find tangent line of $y^2 = \frac{x^2}{xy-4}$ at $(4,2)$

Diff. w.r.t. x: $$2y\frac{dy}{dx} = \frac{2x(xy-4) - x^2\left(y + x\cdot\frac{dy}{dx}\right)}{(xy-4)^2}$$

Substitute $(x,y)=(4,2)$:

$$2(2)\cdot\frac{dy}{dx} = \frac{2\cdot4(4\cdot2-4) - 4^2\left(2+4\frac{dy}{dx}\right)}{(4\cdot2-4)^2}$$

$$\frac{dy}{dx} = 0$$

$\therefore$ eqn: $y=2$

Q29 a) Find the equation of the tangent line to

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \quad \text{at } (x, y)$$

b) Are there any points where the slope is not defined?

Ans: a) Diff:

$$\frac{2x}{25} + \frac{2y}{9} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{25} \Big/ \frac{2y}{9} = -\frac{9}{25} \cdot \frac{x}{y}$$

b) The slope is not well-defined when $y = 0$

Plug $y=0$ into $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$$\Rightarrow x = \pm 5$$

$\therefore$ At $(5,0)$, $(-5,0)$, the slope is not defined.