Week 7

3.4 Chain rule

Consider $f(\underbrace{g(x)}_{z})$

let $z = g(x)$

$y = f(z) = f(g(x))$

chain rule: $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$

Q16)

$f(t) = te^{5-2t}$

$f'(t) = e^{5-2t} + te^{5-2t}(-2)$

$= e^{5-2t}(1-2t)$

Ans

Q4: $\frac{-x}{\sqrt{1-x^2}}$

Q22: $\frac{3s^2}{2\sqrt{s^3+1}}$

Q31)

$y = \left(\frac{x^2+2}{3}\right)^2$

two ways to do:

first way $y = \frac{x^4+4x^2+4}{9}$

$y' = \frac{4x^3+8x}{9}$

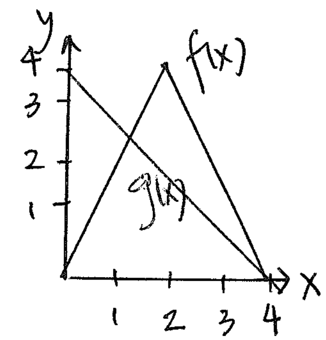
Second way $y' = 2\left(\frac{x^2+2}{3}\right) \cdot \left(\frac{2x}{3}\right)$

$= \frac{4x^3+8x}{9}$

Q43) $y = \sqrt{e^{-3t^2}+5}$

$$y' = \frac{1}{2}\left(e^{-3t^2}+5\right)^{-\frac{1}{2}} \cdot \left(e^{-3t^2}\right) \cdot (-6t)$$

$$= \frac{-3te^{-3t^2}}{\sqrt{e^{-3t^2}+5}}$$

Q51) 

$h(x) = f(g(x))$

Find: a) $h'(1)$, b) $h'(2)$, c) $h'(3)$

a) $h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(3) \cdot g'(1)$

$= (-2)(-1) = 2$

b) $h'(2) = f'(g(2)) \cdot g'(2)$

$= \underbrace{f'(2)}_{\text{donot exist}} \cdot g'(2)$

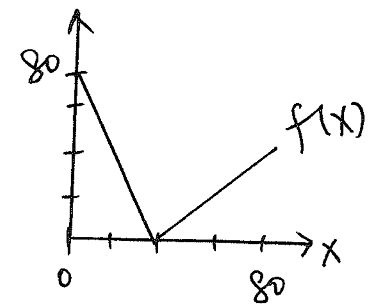
$\therefore h'(2)$ do not exist

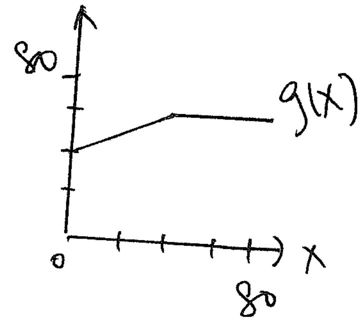
c) $h'(3) = f'(g(3)) \cdot g'(3)$

$= f'(1) \cdot g'(3)$

$= 2(-1) = -2$

57)





$$\frac{d}{dx} g(f(x))\Big|_{x=30}$$

$$= g'(f(30)) \cdot f'(30)$$

$$= g'(20) \cdot f'(30$$

$$= \frac{1}{2} \cdot (-2) = -1$$

61) For what values of x is the graph of $y = e^{-x^2}$ concave down?

want $y'' < 0$

$$y' = -2xe^{-x^2}$$

$$y'' = -2e^{-x^2} + 4x^2 e^{-x^2}$$

$$y'' < 0$$

$$2e^{-x^2}(-1 + 2x^2) < 0$$

i.e. $-1 + 2x^2 < 0$

$$\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

70) given $F(2)=1$, $F'(2)=5$, $F(4)=3$, $F'(4)=7$
$G(4)=2$, $G'(4)=6$, $G(3)=4$ $G'(3)=8$.

Find

a) $H(4)$ if $H(x)=F(G(x))$

$H(4)=F(G(4))=F(2)=1$

b) $H'(4)$ if $H(x)=F(G(x))$

$H'(x)=F'(G(x))\cdot G'(x)$

$H'(4)=F'(G(4))\cdot G'(4)$

$=F'(2)\cdot G'(4)$

$=5\cdot 6=30$

c) $H(4)$ if $H(x)=G(F(x))$

$H(4)=G(F(4))=G(3)=4$

d) $H'(4)$ if $H(x)=G(F(x))$

$H'(x)=G'(F(x))\cdot F'(x)$

$H'(4)=G'(F(4))\cdot F'(4)=G'(3)\cdot F'(4)=(8)(7)=56$

e) $H'(4)$ if $H(x)=\dfrac{F(x)}{G(x)}$

$$H'(x)=\frac{F'(x)\cdot G(x)+G'(x)\cdot F(x)}{(G(x))^2}$$

$$H'(4)=\frac{2\cdot 7-3\cdot 6}{2^2}=-1$$