2.4 Interpretations of the derivative

$$\frac{dy}{dx} \approx \frac{\text{Difference in } y\text{-value}}{\text{Difference in } x \text{ value}}$$

2.5 The second derivative

- derivative of first derivative $\qquad \frac{d}{dx}\left(\frac{d}{dx}f(x)\right) \triangleq f''(x)$

Recall $\quad f \uparrow \Leftrightarrow f'$ pos

$\qquad\qquad f \downarrow \Leftrightarrow f'$ neg.

Now for f'' $\quad$ $+ +$ $\quad$ $- -$

$\qquad\qquad\qquad\quad \cup \qquad\quad \cap$

concave up $\quad$ concave down

2.6 Differentiability

Problem: derivative involve taking limit but limit maynot exist as in Ch.1

$\therefore$ f is differentiable at x

if $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ exists

3.1 Power and Polynomials

If c is a const.

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Hw

Q20: Ans $= -\frac{1}{3}Q^{-4/3}$

Q22: Ans $= ex^{e-1}$

Q50: Ans: $g'(x) = \pi x^{\pi-1} + \pi x^{-(\pi+1)}$ by power and sum rules

Q52: Ans: we cannot write $\frac{1}{3x^2+4}$ as the sum of powers of x multiplied by constants.

Q54: Ans:

$$y' = 3x^2 - 18x - 16$$

$$5 = 3x^2 - 18x - 16$$

$$\Rightarrow x = -1 \text{ or } x = 7$$

when $x=-1$, $y=7$

when $x=7$, $y=-209$

$\therefore$ the two points are $(-1,7)$, $(7,-209)$

Q55 Find the equation of the line tangent to the graph of f at (1,1), where $f(x) = 2x^3 - 2x^2 + 1$

Ans: $f'(x) = 6x^2 - 4$

At $x=1$, $f'(1) = 6 - 4 = 2$

$\therefore$ the eqn for tangent line is $2 = \frac{y-1}{x-1}$

$\Rightarrow y = 2x - 1$

Q59 For what values of x is the graph of $y = x^5 - 5x$ both increasing & concave up?

Ans: $f \uparrow \Rightarrow f'$ pos

$f \cup \Rightarrow f''$ pos

$\therefore$ for $f(x) = x^5 - 5x$

$$\begin{cases} f'(x) = 5x^4 - 5 > 0 \Rightarrow x < -1 \text{ or } x > 1 \\ f''(x) = 20x^3 > 0 \Rightarrow x > 0 \end{cases}$$

$\Rightarrow x > 1$

3.2 The exponential function

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = (\ln a)a^x$$

Q5 $\quad y = 5x^2 + 2^x + 3$

$\Rightarrow y' = 10x + (\ln 2)(2^x)$

Q15 $\quad h(z) = (\ln 2)^z$

$h'(z) = (\ln(\ln 2))(\ln 2)^z$

Q25 $\quad y(x) = a^x + x^a$, a const.

$\Rightarrow y'(x) = (\ln a)a^x + ax^{a-1}$

Q44 a) Find the slope of the graph of $f(x) = 1 - e^x$ at the point where it crosses the x-axis.

b) Find the eqn of the tangent line to the curve at this point.

c) Find the eqn of the line perpendicular to the tangent line at this point.

Ans a) When it crosses x-axis, $y=0$

$\therefore f(x) = 1 - e^x = 0 \quad \Rightarrow x = 0$

& $f'(x) = -e^x$

$\therefore$ slope of tangent $= f'(0) = -e^0 = -1$

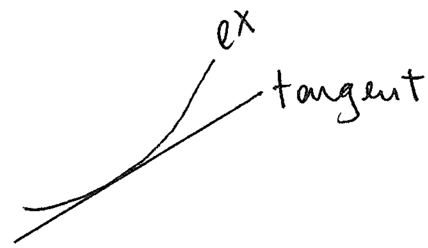
b) $-1 = \frac{y-0}{x-0} \Rightarrow y = -x$

c) slope $= 1$

$\therefore$ eqn $\Rightarrow y = x$

Q47 Show $e^x \geq 1+x$ by eqn of tangent of e^x at $x=0$

Ans: let $f(x) = e^x$



& $f'(x) = f''(x) = e^x > 0$

$\therefore f(x) >$ tangent line at 0.

where $f'(0) = e^0 = 1$

$\therefore$ eqn of tangent at $x=0$ is $1 = \frac{y-1}{x}$

$\Rightarrow y = 1+x$

$\therefore e^x > 1+x$

3.3 Product & Quotient Rules

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

Q9 $f(y) = 4^y(2-y^2)$

$$f'(y) = (\ln 4)\,4^y(2-y^2) + (-2y)4^y$$

Q27 $w(x) = \dfrac{17e^x}{2^x}$

$$w'(x) = \frac{17e^x \cdot 2^x - 2^x(\ln 2)\cdot 17e^x}{(2^x)^2}$$

$$= \frac{17e^x - 17e^x \cdot \ln 2}{2^x}$$

Q44 For what intervals is $g(x) = \dfrac{1}{x^2+1}$ concave down?

Ans: Recall $\frown$

$$\therefore\ g'(x) = \frac{-2x}{(x^2+1)^2} = \frac{-2x}{x^4+2x^2+1}$$

$$g''(x) = \frac{-2(x^2+1)^2 - (4x^3+4x)(-2x)}{(x^2+1)^4}$$

$$= \frac{2(3x^2-1)}{(x^2+1)^3}$$

want $g''(x) < 0$

$$\frac{2(3x^2-1)}{(x^2+1)^3} < 0$$

important: since $(x^2+1)^3 > 0$ for all x !

$$3x^2 - 1 < 0$$

$$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Q49 If $H(3)=1$, $H'(3)=3$, $F(3)=5$, $F'(3)=4$.

Find a) $G'(3)$ if $G(z)=F(z)\cdot H(z)$

b) $G'(3)$ if $G(w)=\dfrac{F(w)}{H(w)}$

Ans: a) $G'(z)=F'(z)\cdot H(z)+F(z)\cdot H'(z)$

$G'(3)=F'(3)\cdot H(3)+F(3)\cdot H'(3)$

$=4\cdot 1+3\cdot 5=19$

b) $$G'(w)=\frac{F'(w)\cdot H(w)-F(w)\cdot H'(w)}{(H(w))^2}$$

$=-11$