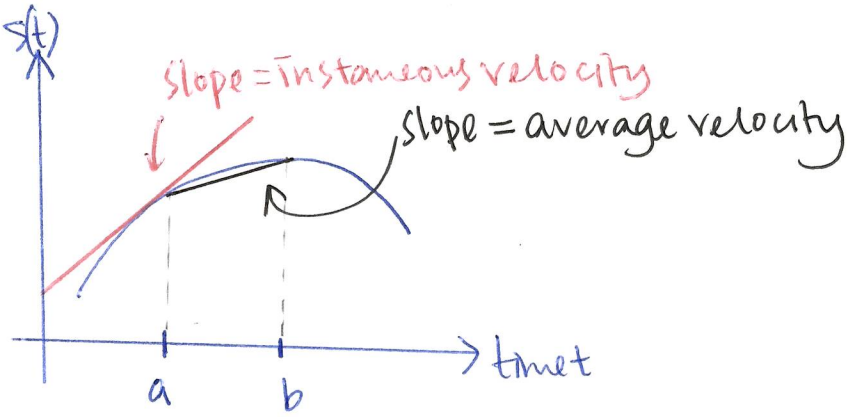
MATH 10A - Week 4

§2.1 How do we measure speed

Let $s(t)$ be position at time t

$$\text{average velocity between } a \le t \le b = \frac{s(b)-s(a)}{b-a}$$

$$\text{instantaneous velocity at } t=a = \lim_{h \to 0} \frac{s(a+h)-s(a)}{h}$$



Hw:

Hint for 9(b): We see in part (a) that as we choose a smaller and smaller interval around $t=1$, the average velocity appears to get closer to 8,

So estimate the instantaneous velocity at $t=1$ to be 8 m/sec.

21 The graph of f(t) gives the position of a particle at the time t. List the following from smallest to largest

A: average velocity between $t=1$ & $t=3$.

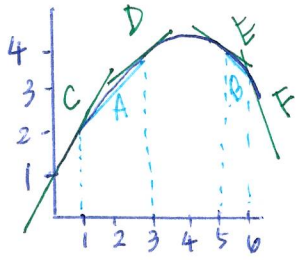
B: " " $t=5$ & $t=6$

C: instantaneous velocity at $t=1$

D: " " at $t=3$

E: " " at $t=5$

F: " " at $t=6$



Ans: $F<B<E<O<D<A<C$

Q23: Find the average velocity over the interval $0 \le t \le 0.2$ & estimate the velocity at $t = 0.2$

t (sec)	0	0.2	0.4	0.6	0.8	1.0
s (ft)	0	0.5	1.8	3.8	6.5	9.6

Ans:

$$\underset{0<t<0.2}{\text{average velocity}} = \frac{0.5}{0.2} = 2.5 \text{ ft/sec}$$

$$\underset{0.2<t<0.4}{\text{average velocity}} = \frac{1.8-0.5}{0.4-0.2} = 6.5 \text{ ft/sec}$$

$$\therefore \text{estimate} = \frac{1}{2}(6.5+2.5) = 4.5 \text{ ft/sec}$$

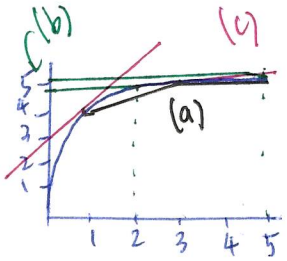
§2.2 The derivative at a point

$$\text{derivative of } f \text{ at } a = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Q11: Which is larger?

a) Average rate of change:
between $x=1.3$ or between $x=3.5$?

b) $f(2)$ or $f(5)$?



c) $f'(1)$ or $f'(4)$?

Q12. Suppose $f(100)=35$ and $f'(100)=3$

Estimate $f(102)$

Ans:

$$f'(100)=\lim_{h\to 0}\frac{f(100+h)-f(100)}{h}$$

Put $h=2$

$$\approx \frac{f(102)-f(100)}{2}$$

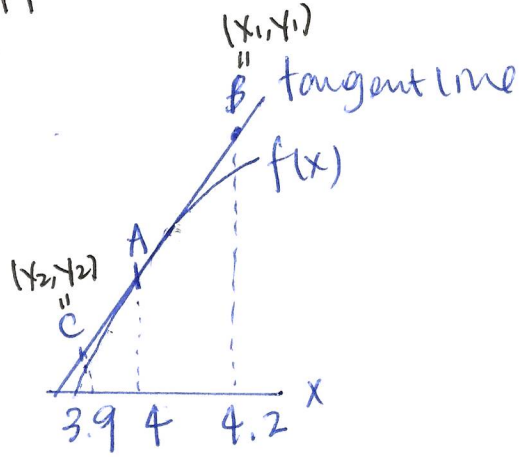
$$\therefore \quad 3\approx\frac{f(102)-35}{2}$$

$$f(102)\approx 41$$

Q13 $f(4)=25$

$f'(4)=1.5$

Find A. B. C



Ans: $A=(4,25)$

For B: $\frac{y_1-25}{4.2-4}=1.5 \Rightarrow y_1=25.3$

$B=(4.2,25.3)$

C: $\frac{y_2-25}{3.9-4}=1.5 \Rightarrow y_2=24.85$

$C=(3.9,24.85)$

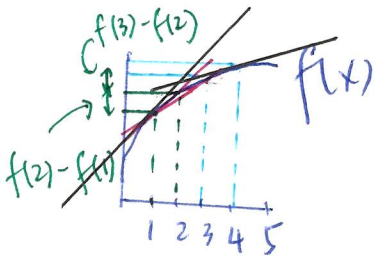
(b) Decide which is larger. Explain.

a) $f(3)$ or $f(4)$

b) $f(3)-f(2)$ or $f(2)-f(1)$

c) $\frac{f(2)-f(1)}{2-1}$ or $\frac{f(3)-f(1)}{3-1}$

d) $f'(1)$ or $f'(4)$



Ans:

a) Since f is increasing. $f(4) > f(3)$.

b) From the graph, $f(2)-f(1) > f(3)-f(2)$

c) $\frac{f(2)-f(1)}{2-1} > \frac{f(3)-f(1)}{3-1}$

Since slope of the secant line joining the point of $X=1$ & $X=2$ is larger than that of $X=1,3$

d) $f'(1) > f'(4)$

The f is steeper at $t=1$ that $t=4$

2.3 Derivative Functions

If $f' > 0$, then f is increasing

If $f' < 0$, then f is decreasing.

- Derivative of a const. function.

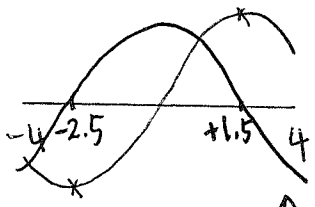
  If $f(x) = k$, then $f'(x) = 0$

- Derivative of a linear function

  If $f(x) = b + mx$, then $f'(x) = m = \text{slope}$.

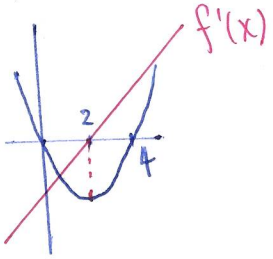
- If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

Q5 Graph the derivative



↑ answer.

Q29 Sketch f'(x)



Since f(x) is a poly. of degree 2

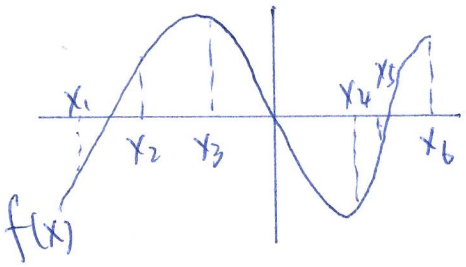
Q40 At which of the labeled x-value is

a) f(x) greatest?

b) f(x) least?

c) f'(x) greatest?

d) f'(x) least



a) x_3

b) x_4

c) x_5

d) x_3

Q41. Graph of f'

On what intervals is f

a) increasing

$x_1 < x < x_3$

b) decreasing

$0 < x < x_1$, $x_3 < x < x_5$