WEEK 3

§1.6 Powers, Polynomials, & Rational function

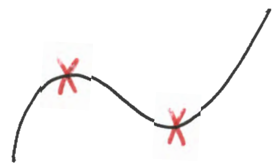
Polynomials

degree

$$y = p(x) = a_n x^{n} + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

n th degree poly "turn around" at most n-1 times

e.g. $n=3$ ⇝ turn at most 2 times



graphing rational function: $f(x) = \dfrac{a_n x^n + \cdots + a_0}{b_m x^m + \cdots + b_0} = \dfrac{p(x)}{q(x)}$

Factorize!

- look at bad points give you $\frac{1}{0}$!

Ending behavior!

- <u>secretly</u> plug in $x = \pm\infty$

3 cases: $\dfrac{\text{constant}}{\infty} = 0$, $\dfrac{\infty}{\text{constant}} = \infty$. $\dfrac{\infty}{\infty}$

For $\frac{\infty}{\infty}$ case, look at highest degree in $p(x), q(x)$

i.e. think $\dfrac{a_n x^n}{b_m x^m} = \dfrac{a_n}{b_n} x^{n-m}$

and check again

Q3 Determine the end behavior of each function as $x \to +\infty$, $x \to -\infty$

a) $f(x) = x^7$

as $x \to \infty$. $f(x) \to \infty$

as $x \to -\infty$ $f(x) \to -\infty$

b) $f(x) = 3x + 7x^3 - 12x^4$

highest degree

as $x \to \infty$ (think: $3\infty + 7\infty - 12\infty$). $f(x) \to -\infty$

$x \to -\infty$. (still: $3\infty - 7\infty - 12\infty$). $f(x) \to -\infty$

c) $f(x) = x^{-4}$

$= \frac{1}{x^4}$

as $x \to \infty$ (think $\frac{1}{\infty}$). $f(x) \to 0$

$x \to -\infty$ $\left(\frac{1}{(-\infty)^4} = \frac{1}{\infty}\right)$. $f(x) \to 0$

d) $f(x) = \frac{6x^3 - 5x^2 + 2}{x^3 - 8}$

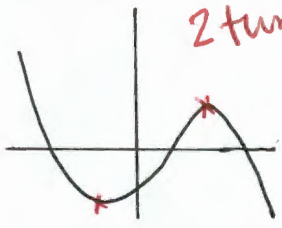
$\frac{\infty}{\infty}$ case.

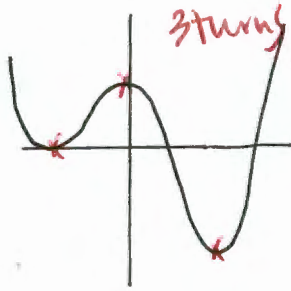
consider $\frac{6x^3}{x^3} = 6$

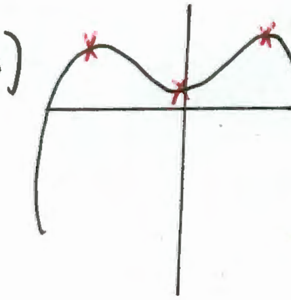
$\therefore$ as $x \to \infty$. $f(x) \to 6$

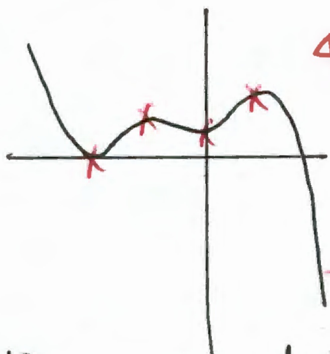
$x \to -\infty$ $f(x) \to 6$

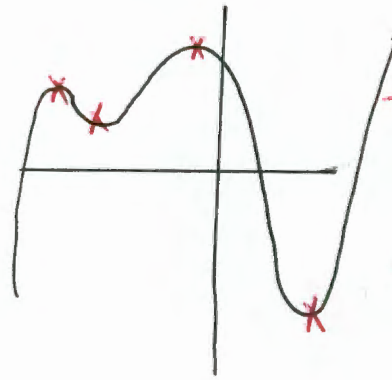
Q6 a) What is the min possible degree?
b) Is the leading poly +ve or −ve?

(I)

2 turns
−ve

(II)

3 turns
+ve

(III)

3 turns
−ve

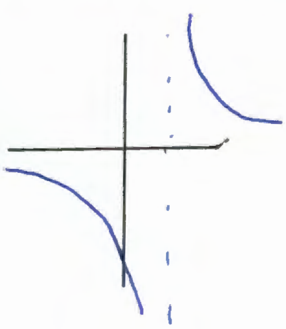
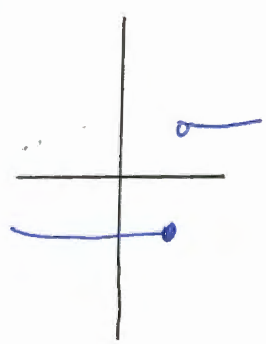
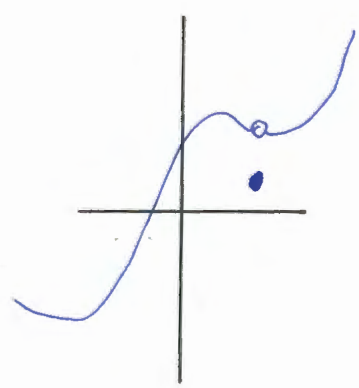
(IV)

4 turns
−ve

(V)

+ve
4 turns

Min possible degree
(i) 3 (ii) 4 (iii) 4 (iv) 5 (v) 5

§1.7 Continuity
- think: can go along the graph without problem!

∴ NOT CONTINUOUS

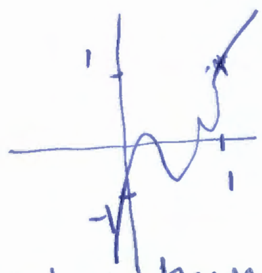
Q11 Show there is a numbe c with $0 \le c \le 1$ s.t. $f(c) = 0$

$f(x) = x^3 + x^2 - 1$

$f(x)$ is a polynomial $\Rightarrow$ continuous

$f(0) = -1 < 0$

$f(1) = 1 > 0$

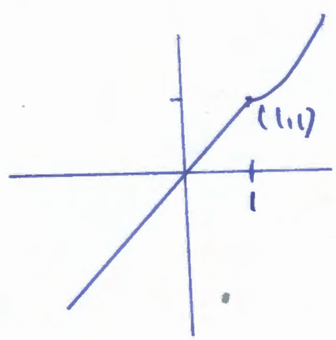


By intermediate-value thm, $\exists\, c$, $0 \le c \le 1$ s.t.

$f(c) = 0$

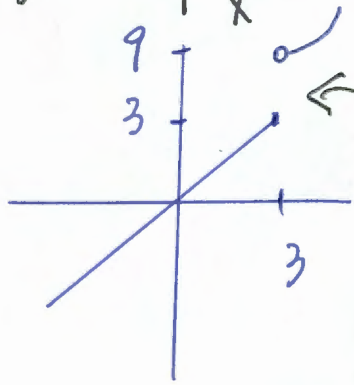
Q15 Is it continuous?

a) $f(x) = \begin{cases} x & x \le 1 \\ x^2 & 1 < x \end{cases}$



continuous (since $1 = 1^2$)

b) $g(x) = \begin{cases} x & x \le 3 \\ x^2 & 3 < x \end{cases}$



not continuous (since $3 \neq 3^2$)

Q22 If possible, choose k s.t. f is continuous

$$f(x) = \begin{cases} \dfrac{5x^3-10x^2}{x-2} & x \neq 2 \\ k & x=2 \end{cases}$$

Only problem @ $x=2$.

Want $\dfrac{5(2)^3-10(2)^2}{2-2} = k$

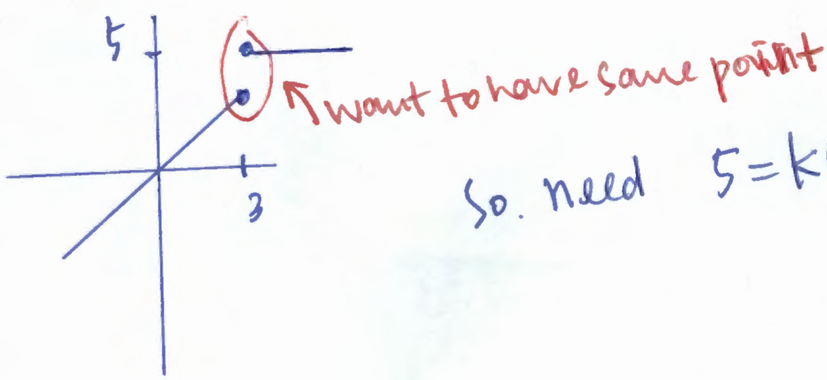
problem

But $\dfrac{5x^3-10x^2}{x-2} = \dfrac{5x^2(x-2)}{x-2} = 5x^2 \to 20$ as $x \to 2$.

$\therefore$ if $k=20$. it is continuous

Q19. Choose k s.t. f is continuous

$$f(x) = \begin{cases} kx & x \leq 3 \\ 5 & 3 < x \end{cases}$$



So. need $5 = k(3) \Rightarrow k = \frac{5}{3}$

§1.8 Limits

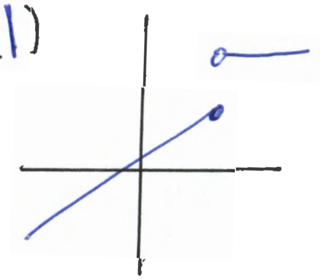
when $x \to c$, think how $f(x) \to$?

if there exists L s.t. $f(x) \to L$, limit exist.

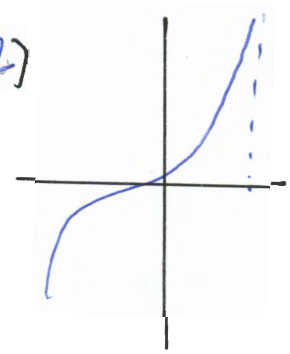
NOT exist: (1) left limit $\neq$ right limit

(2) get to infinity

(1)

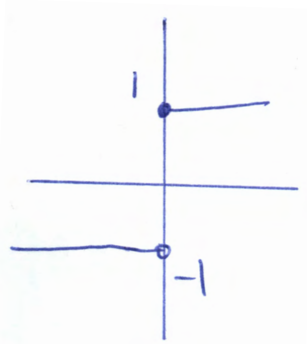


(2)



Q4 Estimate limit graphically

$$\lim_{x \to 0} \frac{|x|}{x}$$



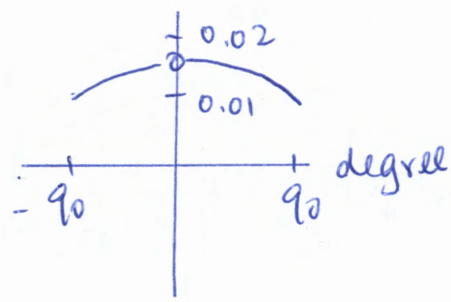
$x < 0$, $\frac{|x|}{x} = \frac{-x}{x} = -1$

$x \geq 0$, $\frac{|x|}{x} = \frac{x}{x} = 1$

DO NOT EXIST

Q9 Use a graph to estimate (use degree)

$$\lim_{\theta \to 0} \frac{\sin \theta^\circ}{\theta^\circ}$$



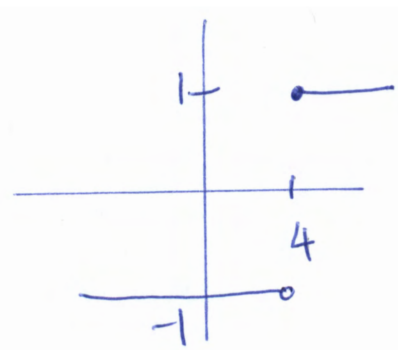
Q11 Evaluate $\lim_{x \to a^+} f(x)$, $\lim_{x \to a^-} f(x)$, $\lim_{x \to a} f(x)$

$a = 4$, $f(x) = \frac{|x-4|}{x-4}$

$a > 4$. $f(x) = \frac{x-4}{x-4} = 1 \Rightarrow \lim_{x \to 4^+} f(x) = 1$

$x < 4$ $f(x) = \frac{-(x-4)}{x-4} = -1 \Rightarrow \lim_{x \to 4^-} f(x) = -1$

$\therefore \lim_{x \to a} f(x)$ does NOT exist



· For Q39, Q43. find limit as $x \to \infty$

· Q 39. $$f(x) = \frac{x^2+2x-1}{3+3x^2}$$

$\frac{\infty}{\infty}$ case

$$\frac{x^2+2x-1}{3+3x^2} \simeq \frac{1+\frac{2}{x}-\frac{1}{x}}{\frac{3}{x^2}+3}$$

as $x \to \infty$, $\frac{1}{x}$, $\frac{3}{x^2}$ terms $\to 0$.

$$\therefore \lim_{x \to \infty} \frac{x^3+2x-1}{3+3x^2} = \frac{1}{3}$$

Q 43 $$f(x) = \frac{3e^x+2}{2e^x+3}$$

$$f(x) = \frac{3e^x+2}{2e^x+3} = \frac{3+\frac{2}{e^x}}{2+\frac{3}{e^x}}$$

again as $x \to \infty$, $\frac{1}{e^x} \to 0$

$$\therefore \lim_{x \to \infty} f(x) = \frac{3}{2}$$

Q 46 Find k s.t. limit exists

$$\lim_{x \to 4} \frac{x^2-k^2}{x-4}$$

$\frac{\text{Something}}{0}$ case.

$$\frac{x^2-k^2}{x-4} = \frac{(x-k)(x+k)}{x-4}$$ ← want to cancel it

$\therefore$ want $k = \pm 4$

Q 47 $\lim_{x \to 1} \frac{x^2 - kx + 4}{x - 1}$

$\frac{x^2 - kx + 4}{x - 1}$ again want to cancel $x - 1$

$\therefore$ want $x = 1$ is a root of $x^2 - kx + 4$.

$\therefore$ want $x^2 - \cancel{4}kx + 4 = 0$ when $x = 1$

$\therefore$ $(1)^2 - k + 4 = 0$

$5 - k = 0 \Rightarrow k = 5$