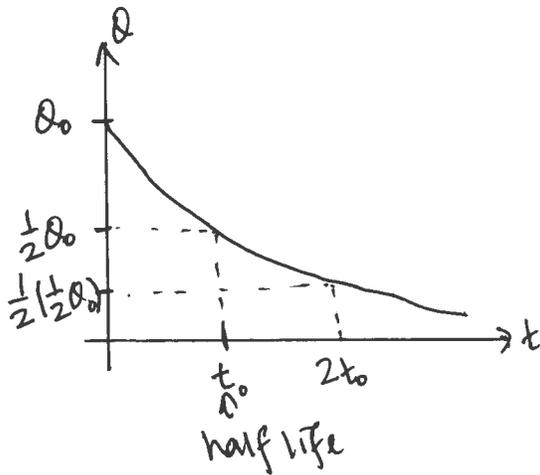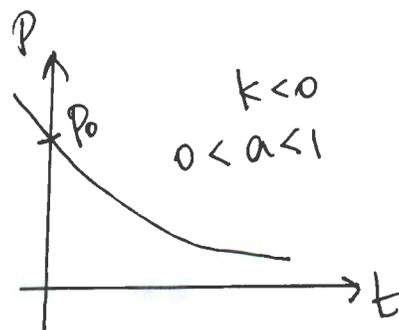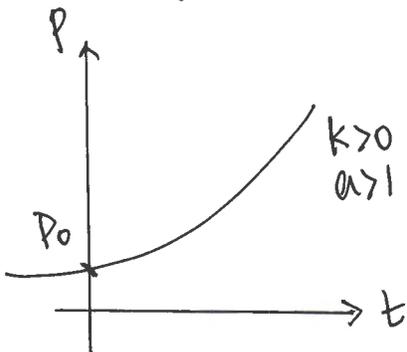Week 2

1.2 Exponential Functions

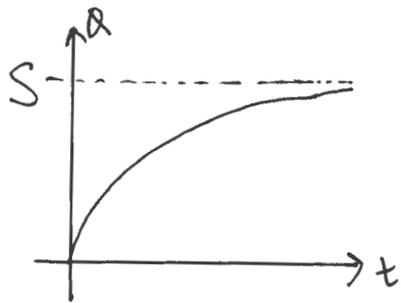- P is an exponential function of t with base a if

$$P = P_0 a^t$$

Half life

The half life of an exponentially decaying quantity is the time required for the quantity to be reduced by a factor of one half.
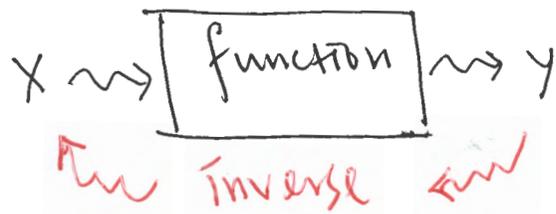

half life

general formula $P = P_0 a^t = P_0 e^{kt}$

$$Q = S(1 - e^{-kt}), \quad k > 0$$

1.3 New Functions from old

$$X \rightsquigarrow \boxed{\text{function}} \rightsquigarrow y$$

inverse fun

1.4 Logarithmic Functions

$$\boxed{\begin{array}{l} \log(AB) = \log A + \log B \\ \log A^p = p \log A \end{array}}$$

$$\boxed{\begin{array}{lcl} \log_{10} X = c & \Longleftrightarrow & 10^c = X \\ \ln X = c & \Longleftrightarrow & e^c = X \end{array}}$$

Q16. Solve for x using log.

$$10^{x+3} = 5e^{7-x}$$

Sol.

$$10^{x+3} = 5e^{7-x}$$

Apply log

$$\log\left(10^{x+3}\right) = \log\left(5e^{7-x}\right)$$

$\log A^p = p\log A$ $\quad (x+3)\log 10 = \log\left(5e^{7-x}\right)$

$\log 10 = 1$ & $\log(AB) = \log A + \log B$

$$x+3 = \log 5 + \log e^{7-x}$$

$$= \log 5 + (7-x)\log e$$

$$= \log 5 + 7\log e - x\log e$$

$$x(1+\log e) = \log 5 + 7\log e - 3$$

$$x = \frac{\log 5 + 7\log e - 3}{1 + \log e}$$

$$= 0.515$$

Q53) picture contain 99.5% C-14

half life of C-14 = 5730 years.

Sol)
$$\frac{1}{2} = e^{-k(5730)} \qquad (\text{since } Q = Q_0 e^{-kt})$$
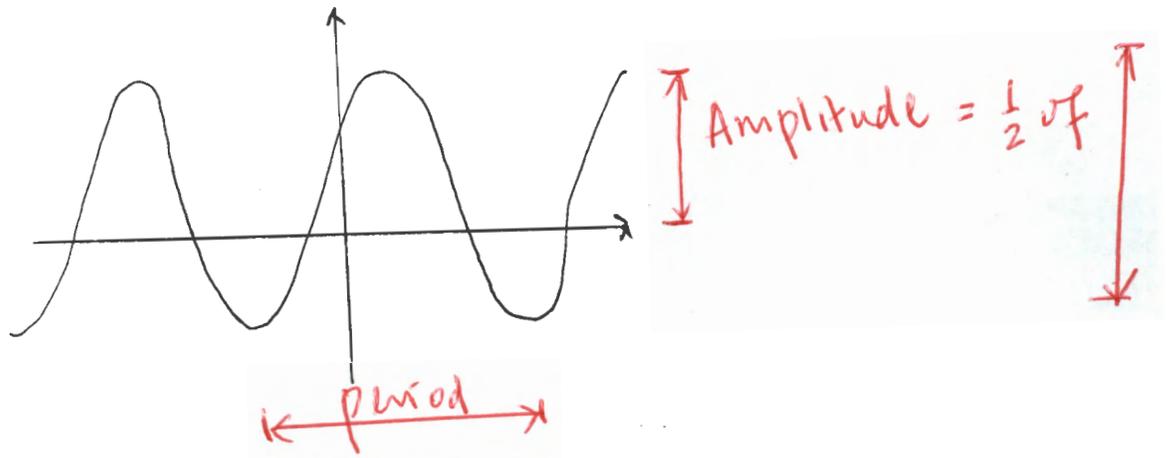
$$\therefore k = 1.21 \times 10^{-4}$$

$\therefore$ We can find time, the age of printing.

$$e^{-1.21 \times 10^{-4} t} = 99.5\%$$

$$t = \frac{\ln 0.995}{-1.21 \times 10^{-4}} = 41.43 \text{ years}$$

Since Vermeer died in 1675, the painting is fake
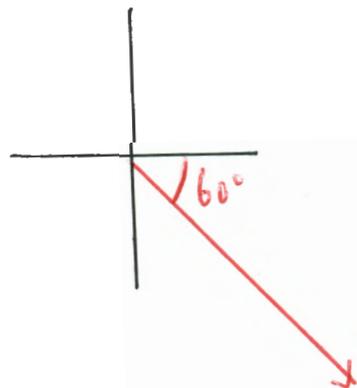
1.5 Trigonmetric Functions

Amplitude = $\frac{1}{2}$ of ↕

period

	0°	30°	45°	60°	90°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

S	A
T	C

Q9. Draw $\overset{rad}{=}-1$ & check sin.cos.tan +ve/-ve/0/undefind

Sol. $-1 \cdot rad = -1\, rad \cdot \frac{180°}{\pi\, rad} = -\frac{180°}{\pi} \approx -60°$

$\cos(-1)$ is positive
$\sin(-1)$ ·· negative
$\tan(-1)$ ·· negative

Q 32. Find a solution if possible
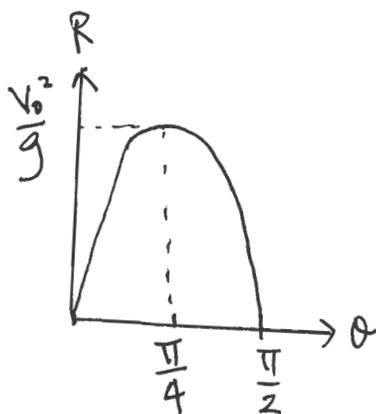
$$8 = 4\sin(5x)$$

Sol.

$$\Rightarrow \sin(5x) = 2$$

Impossible

Q44

$$R = \frac{v_0^2}{g}\sin(2\theta)$$

Sketch R as a function $0 \le \theta \le \frac{\pi}{2}$.
What angle gives the maximum range? What is it?

Sol.

$\sin(2\theta)$ has period $\frac{2\pi}{2} = \pi$

$\therefore \frac{\pi}{4}$ gives max range & max $= \frac{v_0^2}{g}$