PLANS FOR NUMBER THEORY LEARNING HURWITZ SPACE SEMINAR

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Please ask Melanie if you need more details about the intended topic.

(1) September 4, Introduction and organization: global fields, basic questions of arithmetic statistics (field counting, class group counting), the geometric side of function fields, moduli spaces, organization of the seminar
(2) September 11, Moments determining a distribution and Class group counting as field counting [Probably the easiest lecture to give]: introduce moments of random abelian groups [CKL+15, Section 3.3], moments determining a distribution [EVW16, Lemma 8.2], [Woo15, Theorem 3.1] (note we are not yet determining all moments, so this is mostly for philosophy at this point), class group is Galois group of Hilbert class field (Section 3.4 of Poonen's "A brief summary of the statements of class field theory"), surjections from class groups of Γ fields to abelian A corresponding to $A \rtimes \Gamma$ extensions when $(|A|, |\Gamma|) = 1$ ([Woo16, Section 3], [LWZ19, Section 9])
(3) September 18, Grothendieck-Lefschetz trace formula [Ideally given by someone who already knows about etale cohomology]: Lang-Weil bound [LW54, Theorem 1] and how it implies that for $q \to \infty$ with dimension, degree, ambient projective space fixed, $X(\mathbb{F}_q) \sim cq^d$, where c is the number of geometric components of X defined over $\mathbb{F}_q$ and d is the dimension (do an example like $x^2 - Dy^2 = 0$ to see that need geometric components defined over $\mathbb{F}_q$), Brief statement of properties of étale cohomology (comparsion with singular cohomology, Frobenius eigenvalue on H_c^{top}, bound on size of Frob eigenvalues for smooth), statement of Grothendieck-Lefschetz trace formula for smooth schemes (not necessarily proper)
(4) September 25, Introduction to Hurwitz spaces: [FV, Section 1]
(5) October 2, More Hurwitz schemes: [LWZ19]: Lemma 10.2 statement, Section 11 describe functor of points of the schemes $\mathrm{Hur}^n_{G,*}$ and $\mathrm{Hur}^n_{G,c}$, statement of Proposition 11.4 (define map to Conf), Proof of Lemma 10.2 (explain correspondence with function fields), Lemma 11.8
(6) October 9, Algebraic lifting invariants I and components of Hurwitz spaces over the complex numbers, see "An algebraic lifting invariant..." on my webpage, Sections 2,3
(7) October 16, Algebraic lifting invariants II, see "An algebraic lifting invariant..." on my webpage, Sections 4-6
(8) October 30, Example: the 5-part of real quadratic fields for large q. Do [LWZ19, Section 10] for $\Gamma = \mathbb{Z}/2\mathbb{Z}$ and $H = \mathbb{Z}/5\mathbb{Z}$ with the -1 action of Γ (Lemma 10.2 already covered, Lemms 10.3 just state), replace Lemma 10.4 with your own computation of components here, using the algebraic lifting invariant (which you can assume gives exactly components, use end of Theorem 12.1 to see which are Frob fixed), and then sketch the proof of Theorem 1.4 in this case

(9) November 6, Implications of Homological Stability [EVW16]: Main argument in proof of Theorem 8.8 on page 778 (don't worry about what exactly the spaces are here, or the technical details, just show how this kind of bound on cohomology gives what kind of result), Statement of Proposition 7.8, how Proposition 7.8 follows from Corollary 6.2 (main input) and Proposition 2.5
(10) November 13, Homological Stability I [EVW16, Section 3 and start of Section 4] [Coordinate with next lecture], make to to remind which Hurwitz spaces and what hypotheses on (G, c)
(11) November 20, Homological Stability II [EVW16, rest of Section 4] [Coordinate with previous lecture]
(12) December 4, Homological Stability III [EVW16, Section 5]

References

[CKL+15] Julien Clancy, Nathan Kaplan, Timothy Leake, Sam Payne, and Melanie Matchett Wood. On a Cohen–Lenstra heuristic for Jacobians of random graphs. *Journal of Algebraic Combinatorics*, pages 1–23, May 2015.

[EVW16] Jordan S. Ellenberg, Akshay Venkatesh, and Craig Westerland. Homological stability for Hurwitz spaces and the Cohen-Lenstra conjecture over function fields. *Annals of Mathematics. Second Series*, 183(3):729–786, 2016.

[FV] Michael D. Fried and Helmut Völklein. The inverse Galois problem and rational points on moduli spaces. *Mathematische Annalen*, 290(1):771–800.

[LW54] Serge Lang and André Weil. Number of points of varieties in finite fields. *American Journal of Mathematics*, 76:819–827, 1954.

[LWZ19] Yuan Liu, Melanie Matchett Wood, and David Zureick-Brown. A predicted distribution for Galois groups of maximal unramified extensions. *arXiv:1907.05002 [math]*, July 2019.

[Woo15] Melanie Matchett Wood. Random integral matrices and the Cohen Lenstra Heuristics. *arXiv:1504.04391 [math]*, (to appear in American Journal of Math.), April 2015.

[Woo16] Melanie Matchett Wood. Asymptotics for number fields and class groups. In *Directions in Number Theory: Proceedings of the 2014 WIN3 Workshop*. Springer, New York, NY, 1st ed. 2016 edition edition, July 2016.

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