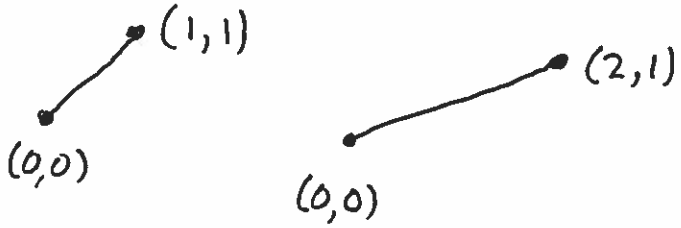
Jake's Ma21a Week10 Worksheet

Vector fields

$\vec{F}_1 = \langle -y, x \rangle$ $\quad \vec{F}_2 = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$

$\vec{F}_3 = \langle 1, x \rangle$ $\quad \vec{F}_4 = \langle 2xy^2 + 3x^2, 2yx^2 \rangle$

Curves



(I) $\text{curl}(\langle P, Q \rangle) = \nabla \times \langle P, Q \rangle = Q_x - P_y$

(II) $\int_C \vec{F} = \int_a^b \vec{F} \cdot \vec{r}'(t)\, dt$ where $\vec{r}(t)$ params C from $a \le t \le b$

(III) $\int_C \nabla f = \int_a^b \vec{\nabla} f \cdot \vec{r}'\, dt = \int_a^b \frac{d}{dt} f\, dt = f(\vec{r}(b)) - f(\vec{r}(a))$

(IV) See next page.

Problems

① Compute $\nabla \times \vec{F}_i$ and $\nabla \times \nabla f$.

② Find potential functions for $\vec{F}_i$ if they exist.

③ Compute $\int_C \vec{F}_1$ $\quad \int_C \vec{F}_3$ $\quad \int_{-C} \vec{F}_3$

④ Compute $\int_D \vec{F}_4$

$$\int_{C_1} \vec{F} - \int_{C_2} \vec{F} = \int_{C_1 - C_2} \vec{F} = 0$$