

Math 221: Problem Set 5

- (1) Let R be a local ring containing with maximal ideal $\mathfrak{m}$, and let $f : M \rightarrow N$ be an R -module homomorphism which induces a surjection $M/\mathfrak{m}M \rightarrow N/\mathfrak{m}N$. Show that if N is finitely generated, then f is a surjection. Give an example to show that f need not be a surjection if N is not finitely generated.
- (2) Let R be a local ring. Show that every finitely generated projective R -module is isomorphic to R^n for some uniquely determined integer $n \geq 0$.
- (3) Let R be a Noetherian ring and let M and N be finitely generated R -modules. Prove that a prime ideal $\mathfrak{p} \subseteq R$ is an associated prime of $\text{Hom}_R(M, N)$ if and only if $\mathfrak{p}$ is an associated prime of N which belongs to the support of M .
- (4) Let R be a commutative ring. We let $R[[x]]$ denote the ring of formal power series $f(x) = \sum_{n \geq 0} a_n x^n$ where each $a_n \in R$. Prove that if R is Noetherian, then $R[[x]]$ is Noetherian.
- (5) Let R be a commutative ring, and suppose that every prime ideal of R is finitely generated. Prove that R is Noetherian.