

Math 221: Problem Set 11

- (1) Let $f : A \rightarrow B$ be a map of commutative rings, let μ denote the multiplication map $\mu : B \otimes_A B \rightarrow B$, and let I be the kernel of μ . Show that the construction

$$b \mapsto 1 \otimes b - b \otimes 1$$

determines an A -linear derivation from B into I/I^2 , which induces an isomorphism of B -modules $\Omega_{B/A} \rightarrow I/I^2$.

- (2) Let k be a field and let $\bar{k}$ be its algebraic closure. Show that $\Omega_{\bar{k}/k} \simeq 0$ (note that $\bar{k}$ need not be a separable extension of k).
- (3) Let R be a complete local Noetherian ring having maximal ideal $\mathfrak{m}$. Assume that the field $R/\mathfrak{m}$ has characteristic zero. Show that there exists a subfield $k \subseteq R$ such that the composite map $\alpha : k \rightarrow R \rightarrow R/\mathfrak{m}$ is an isomorphism. (Hint: use Zorn's lemma to choose k maximal among subfields of R , and use Hensel's lemma to show that α is an isomorphism).
- (4) Let $f : A \rightarrow B$ and $g : A \rightarrow A'$ be maps of commutative rings, and set $B' = B \otimes_A A'$. Construct an isomorphism $\Omega_{B'/A'} \simeq \Omega_{B/A} \otimes_B B'$.