

Math 221: Problem Set 10

- (1) Let R be a Noetherian ring having finite Krull dimension n . Show that the polynomial ring $R[x]$ has Krull dimension $n + 1$.
- (2) Let $f : R \rightarrow R'$ be a map of Noetherian rings which exhibits R' as integral (but not necessarily finite) over R . Show that the Krull dimension of R' is at most the Krull dimension of R .
- (3) Let $f : R \rightarrow R'$ be a map of commutative rings. Assume that R' is Noetherian, and let $\mathfrak{p}$ be a prime ideal of R such that the map $R/\mathfrak{p} \rightarrow R'/\mathfrak{p}R'$ is injective. Prove that there exists a prime ideal $\mathfrak{q} \subseteq R'$ such that $f^{-1}\mathfrak{q} = \mathfrak{p}$.
- (4) Let R be the infinitely generated polynomial ring $\mathbf{C}[\{x_{i,j}\}_{0 \leq i \leq j}]$. For every integer $n \geq 0$, let $\mathfrak{p}_n$ be the ideal in R generated by the elements $\{x_{i,n}\}_{i \leq n}$. Let S denote the multiplicatively closed subset $R - \bigcup_{n \geq 0} \mathfrak{p}_n$ of R . Prove that $S^{-1}R$ is a Noetherian ring of infinite Krull dimension.