

Math 221: Final Exam

- (1) Let $f : R \rightarrow R'$ be a flat map of commutative rings, and suppose that f induces a surjection $F : \mathbf{Spec} R' \rightarrow \mathbf{Spec} R$. Show that F is a quotient map: that is, a subset $U \subseteq \mathbf{Spec} R$ is open if and only if $F^{-1}U$ is an open subset of $\mathbf{Spec} R'$.
- (2) Let R be a local Noetherian ring, and let $R^\vee$ denote the completion of R (with respect to its maximal ideal). Show that the depth of R coincides with the depth of $R^\vee$.
- (3) Let R be a local Noetherian ring. Show that if R is Cohen-Macaulay, then there exists a (nonzero) finitely generated R -module M whose length and projective dimension are both finite.
- (4) Let R be a local Noetherian ring which contains the field $\mathbf{Q}$ of rational numbers, let G be a finite group acting on R , and let $R^G \subseteq R$ be the fixed points for the action of G . Show that R^G is also a local Noetherian ring, which is Cohen-Macaulay if R is Cohen-Macaulay.