

# Math 112, Problem Set 3 (due Tuesday, February 21)

February 16, 2012

- (1) Regard the plane  $\mathbb{R}^2$  as a metric space with respect to the usual Euclidean distance:

$$d((x, y), (x', y')) = \sqrt{(x - x')^2 + (y - y')^2}.$$

Prove that  $\mathbb{R}^2$  is connected.

- (2) Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be the multiplication function, given by  $f(x, y) = xy$ . Prove that  $f$  is continuous.
- (3) Give an example of a continuous map  $f : X \rightarrow Y$  between metric spaces and a Cauchy sequence  $x_1, x_2, \dots$  in  $X$  such that  $f(x_1), f(x_2), f(x_3), \dots$  is not a Cauchy sequence in  $Y$ .
- (4) Let  $X$  be a metric space, and let  $K, K' \subseteq X$  be compact subsets of  $X$ . Show that if  $K \cap K' = \emptyset$ , then there exists a positive real number  $\epsilon$  with the following property: for every pair of points  $x \in K$  and  $x' \in K'$ , we have  $d(x, x') > \epsilon$ .