MATH21B – MIDTERM 2 REVIEW
SPRING 2018, HARVARD UNIVERSITY

1. Some questions you might have

1.1. Gram-Schmidt, QR-decompositions and orthogonal matrices.

- ☐ What is an orthogonal matrix?
- ☐ What is an upper-triangular matrix?
- ☐ What is the geometric meaning of a matrix being orthogonal?
- ☐ What are standard examples of orthogonal matrices?
- ☐ What is the Gram-Schmidt procedure?
- ☐ What is the goal of the Gram-Schmidt procedure?
- ☐ What is a QR-decomposition?
- ☐ Why would you want to find a QR-decomposition?
- ☐ What is the relationship between Gram-Schmidt and QR-decompositions?

1.2. Least squares and data fitting.

- ☐ What is the least squares solution to a system of linear equations?
- ☐ How do I find the least squares solution?
- ☐ How are least squares solutions used in data fitting?
- ☐ What is the relationship between least squares solutions and inconsistency?
- ☐ When is the least solution an actual solution?

1.3. Determinants.

- ☐ What is the determinant of a matrix?
- ☐ What is the relationship between determinants and invertibility?
- ☐ What is the geometric interpretation of the determinant?
- ☐ When can you geometrically find the determinant?
- ☐ What properties does the determinant have?
- ☐ What is the Leibniz formula for the determinant? What is an up-crossing?
- ☐ What is Laplace expansion? How do you find the signs in the Laplace expansion?
- ☐ What is the determinant of a partitioned matrix (one with blocks)?
- ☐ How do you use row reduction to compute determinants?
- ☐ How does the determinant change when you swap rows or columns? How does it change when we scale a row or column?
- ☐ How do you decide which method to use to compute a determinant?
- ☐ What is the determinant of an upper-triangular or lower-triangular matrix?
- ☐ What is the relationship between determinants and eigenvalues?
- ☐ What is the determinant of a product of matrices?
- ☐ What is the determinant of the inverse of a matrix?

1.4. Characteristic polynomials and the trace.

- ☐ What is the characteristic polynomial?
- ☐ How does the characteristic polynomial relate to the trace?
- ☐ How does the characteristic polynomial relate to the determinant?
- ☐ What is the trace?
- ☐ What properties does the trace have?

- ☐ Why would you want to compute the characteristic polynomial?

1.5. **Eigenvalues and eigenvectors.**

- ☐ What are eigenvalues?
- ☐ What is the geometric interpretation of eigenvalues?
- ☐ How do you find eigenvalues?
- ☐ Why are the eigenvalues the roots of the characteristic polynomial?
- ☐ What is an eigenspace?
- ☐ What is the algebraic multiplicity of an eigenvalue?
- ☐ What is the geometric multiplicity of an eigenvalue?
- ☐ What is the relationship between the algebraic and geometric multiplicity?
- ☐ What is a complex number? Why would we use them?
- ☐ What are complex eigenvalues?
- ☐ Why is there always a complex eigenvalue?
- ☐ What is the relationship between eigenvalues and the determinant?
- ☐ What is the relationship between eigenvalues and the trace?
- ☐ What are the eigenvalues of the inverse of a matrix?
- ☐ What are the eigenvalues of a power of a matrix?
- ☐ What is an eigenvector?
- ☐ What is an eigenspace?
- ☐ What is an eigenbasis?
- ☐ How do I find eigenvectors?
- ☐ How do I find the eigenvalues and eigenvectors of a circular/circulant matrix?
- ☐ What can you say about the eigenvalues of standard matrices such as rotations, reflections, dilations, and projections?

1.6. **Similarity and diagonalizibility.**

- ☐ What does it mean for two matrices to be similar?
- ☐ What is the relationship between similarity and changing bases?
- ☐ How can you tell two matrices are not similar?
- ☐ What does it mean for a matrix to be diagonalizable?
- ☐ Why would you want to diagonalize a matrix?
- ☐ What is the relationship between diagonalization and changing bases?
- ☐ What is the relationship between similarity and diagonalizibility?
- ☐ What numbers appear on the diagonal when you diagonalize a matrix?
- ☐ Why is diagonalizing a matrix the same as finding an eigenbasis?

1.7. **Discrete dynamical systems and stability.**

- ☐ What is a discrete dynamical system?
- ☐ How do you find the matrix for a recursion?
- ☐ What is asymptotic stability?
- ☐ When is a matrix asymptotically stable?

1.8. **Symmetric matrices.**

- ☐ What is a symmetric matrix?
- ☐ What are standard examples of symmetric matrices?
- ☐ What properties does a symmetric matrix have?
- ☐ Why does a symmetric matrix have real eigenvalues?
- ☐ Why are eigenvectors for different eigenvalues of a symmetric matrix orthogonal?
- ☐ Can symmetric matrix always be diagonalized using an orthonormal eigenbasis?
- ☐ What is the spectral theorem?