

**MATH21B – MIDTERM 1 REVIEW**  
**SPRING 2018, HARVARD UNIVERSITY**

1. SOME QUESTIONS YOU MIGHT HAVE

1.1. **Matrices and systems of linear systems of equations.**

- What is a matrix?
- How do you multiply two matrices?
- When can you multiply two matrices?
- What is an augmented matrix?
- How do you find the augmented matrix corresponding to a linear system?
- How do you recognize that a matrix is in reduced row echelon form?
- How do you row reduce a matrix?
- How do you recognize that a system of linear equations is consistent?
- How do you recognize that a system of linear equations has infinitely many solutions?
- What is the identity matrix?
- What is the zero matrix?
- Is there a matrix  $A$  such that  $A^2 = 0$ ?
- What is an invertible matrix?
- When is a matrix invertible?
- How do you find the inverse of a matrix?
- What is the rank of a matrix?
- What is the nullity of a matrix?

1.2. **Linear spaces.**

- What is  $\mathbb{R}^n$ ?
- What does the notation  $\{\vec{v} \mid A\vec{x} = \vec{b}\}$  mean?
- What is a linear subspace?
- What is a linear space?
- How do you check that a subset of a linear space is a linear subspace?
- What are examples of linear subspaces?
- Why is  $C^\infty(\mathbb{R})$  a linear space?
- What is  $\vec{0}$  in  $C^\infty(\mathbb{R})$ ?

1.3. **Spanning sets, linear independence and bases.**

- What is a spanning set?
- What is linear independence or linear dependence?
- How do you check a collection of vectors is linearly independent?
- How do you find a maximal linearly independent subset of a collection of vectors?
- What is a basis?
- How do you find a basis for  $\text{span}(\vec{v}_1, \dots, \vec{v}_n)$ ?
- What is  $\vec{e}_i$ ?
- What is the dimension of a linear space?

**1.4. Images and kernels.**

- What is the kernel  $\ker(A)$ ?
- How do you find a basis for  $\ker(A)$ ?
- What is the relationship between kernels and nullity?
- What is the image  $\text{im}(A)$ ?
- How do you find a basis for  $\text{im}(A)$ ?
- What is the relationship between images and rank?

**1.5. Linear transformations.**

- What is a linear transformation?
- What is the difference between a linear transformation and a matrix?
- How do you find the matrix corresponding to a linear transformation?
- What are examples of linear transformations?

**1.6. Changing basis.**

- What does it mean to change to a different basis?
- Why would you want to change to a different basis?
- How do you write a vector with respect to a new basis?
- How do you write a matrix with respect to a new basis?
- What are examples of bases of  $\mathbb{R}^n$ ?

**1.7. Dot products and orthogonality.**

- How do you compute the length of a vector?
- How do you find the angle between two vectors?
- What does it mean for two vectors to be orthogonal?
- What is the orthogonal complement to a linear subspace?
- How do you find a basis for the orthogonal complement to a linear subspace?
- What is an orthonormal basis?
- How do you find the orthogonal projection onto a linear subspace?