

MATH21B – FINAL REVIEW
SPRING 2018, HARVARD UNIVERSITY

1. SOME QUESTIONS YOU MIGHT HAVE

1.1. **Differential equations in general.**

- What is a differential equation?
- What is an initial condition?
- What is separation of variables?
- How do you make a phase portrait?
- How do you analyze a non-linear system?
- What is an equilibrium and how do you find them?
- What is a null-cline and how do you find them?
- When is an equilibrium stable?
- What is the Jacobian?
- Why would you want to solve a (system of) differential equation(s)?

1.2. **Systems of linear differential equations.**

- What is a system of linear differential equations?
- How do you solve a system of linear differential equations?
- What are the differences between discrete dynamical systems and continuous dynamical systems?
- When is the system $\vec{x}'(t) = A\vec{x}(t)$ stable?

1.3. **The cookbook.**

- How do you solve $x'(t) = \lambda x(t)$?
- How do you solve the harmonic oscillator $x''(t) = -c^2 x(t)$? Why is it called the harmonic oscillator?
- What is a differential operator?
- How do I solve $p(D)x(t) = 0$ for p a polynomial?
- How do I solve $p(D)x(t) = g(t)$?
- What is the operator method?
- What are homogeneous solutions? What is a particular (or inhomogenous) solution?
- What is the heuristic for finding a particular (or inhomogeneous solution)?
- What do you need to memorize from the cookbook?

1.4. **Fourier analysis.**

- What is the linear space C_{per}^∞ of 2π -periodic smooth functions?
- What is the inner product $\langle -, - \rangle$ on C_{per}^∞ ?
- What is special about $\frac{1}{\sqrt{2}}$, $\sin(nx)$, $\cos(nx)$?
- What is a Fourier series? What are Fourier coefficients?
- What is Parseval's identity?
- How do you compute the Fourier series for a function f ?
- What can you say about the Fourier coefficients of an odd or even function?
- Why would you want to compute the Fourier series?

1.5. Partial differential equations.

- What is a partial differential equation?
- What is the heat equation? What are applications of it?
- What is the wave equation? What are applications of it?
- How do you solve the heat equation?
- How do you solve the wave equation?
- What is the general behavior of solutions to the heat equation?
- What is the general behavior of solutions to the wave equation?
- Why do we only see sines and not cosines when solving the heat or wave equation?
- In what sense are solving a system of linear differential equations, solving the heat equation and solving the wave equation analogous?

2. DIFFERENTIAL EQUATIONS YOU SHOULD KNOW HOW TO SOLVE

The following list is non-exhaustive:

- (1) $x'(t) = \lambda x(t)$ with initial condition $x(0)$.
- (2) $x''(t) = -c^2 x(t)$ with initial conditions $x(0)$ and $x'(0)$.
- (3) $\vec{x}'(t) = A\vec{x}(t)$ with initial condition $\vec{x}(0)$.
- (4) $p(D)\vec{x}(t) = g(t)$ with $p(D) = D^2 + bD + c$ and $g(t)$ some combination of polynomials, sines, cosines and exponentials, with initial conditions $x(0)$ and $x'(0)$.
- (5) $f_t(x, t) = \mu f_{xx}(x, t)$ with initial condition $f(x, 0)$.
- (6) $f_{tt}(x, t) = c^2 f_{xx}(x, t)$ with initial condition $f(x, 0)$ and $f_t(x, 0)$.