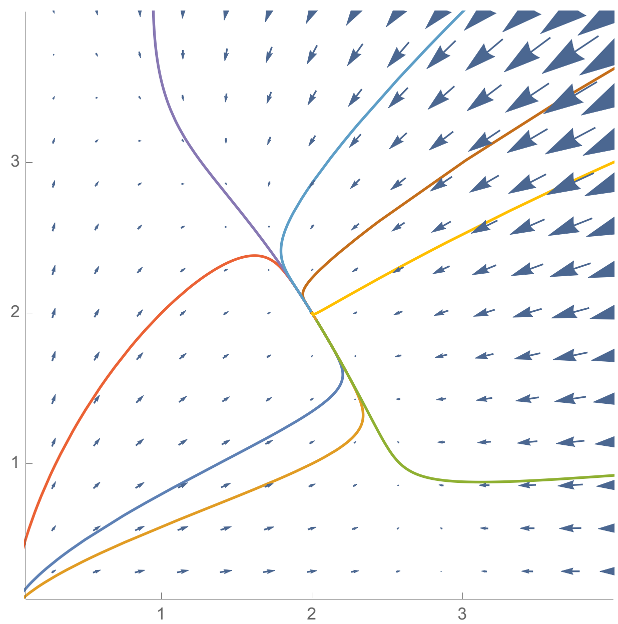
# MATH21B – LECTURE 26: NON-LINEAR SYSTEMS
# SPRING 2018, HARVARD UNIVERSITY

## 1. The Murray population model

**Problem 1.** Analyze the following system of differential equations

$$\frac{dx}{dt} = x(6 - 2x - y)$$
$$\frac{dy}{dt} = y(4 - x - y)$$

for $x \geq 0, y \geq 0$.




## 2. An analysis

**Problem 2.** Consider the following system of differential equations

$$\frac{dx}{dt} = x(y - 1)$$
$$\frac{dy}{dt} = y(2 - x - y)$$

(i) Find the nullclines and equilibria.
(ii) Start to draw a phase portrait by drawing the nullclines, equilibria and a sketch of the vector field by looking at the signs of $f(x, y)$ and $g(x, y)$.
(iii) Is the equilibrium at $(1, 1)$ stable?

**Summary**

- Analyzing a system of differential equations

$$\frac{dx}{dt} = f(x,y)$$
$$\frac{dy}{dt} = g(x,y)$$

is done by following the steps:

(1) Find the *nullclines* $\{(x,y) \mid f(x,y) = 0\}$ and $\{(x,y) \mid g(x,y) = 0\}$.

(2) The horizontal and vertical nullclines intersect in the *equilibria*, those points $(x,y)$ where $f(x,y) = 0$ and $g(x,y) = 0$.

(3) Start drawing a phase portrait by finding the signs of $f(x,y)$ and $g(x,y)$ in the regions bounded by the nullclines.

(4) By linearizing the system near an equilibrium $(a,b)$ we get a system of linear differential equations (the matrix is called the *Jacobian*)

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} \frac{df(a,b)}{dx} & \frac{df(a,b)}{dy} \\ \frac{dg(a,b)}{dx} & \frac{dg(a,b)}{dy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

This describes the behavior near $(a,b)$. In particular, we use it to determine the stability of the equilibria.

(5) Finish drawing the phase portrait by understanding some representative trajectories.