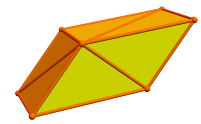
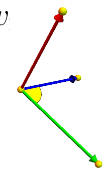
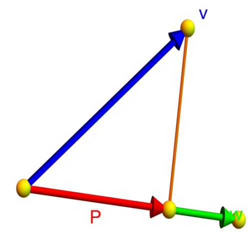
CHECKLIST FIRST HOURLY — Maths21a Summer 2024

Geometry of Space

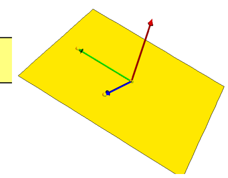


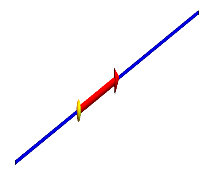




- point $P = (a, b, c)$ with **coordinates** a, b, c.
- vector $\vec{v} = \vec{PQ} = [v_1, v_2, v_3]$ with components v_k.
- unit vector = direction = vector of length 1.
- $\vec{v} = [v_1, v_2, v_3], \vec{w} = [w_1, w_2, w_3], \vec{v} + \vec{w} = [v_1 + w_1, v_2 + w$
- dot product $\vec{v}.\vec{w} = v_1w_1 + v_2w_2 + v_3w_3 = |\vec{v}||\vec{w}|\cos(\alpha)$
- angle $\cos(\alpha) = (\vec{v} \cdot \vec{w})/|\vec{v}||\vec{w}|$.
- cross product $\vec{v} \cdot (\vec{v} \times \vec{w}) = 0$, $\vec{w} \cdot (\vec{v} \times \vec{w}) = 0$
- triple scalar product $\vec{u} \cdot (\vec{v} \times \vec{w})$
- area of parallelogram $|\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}|\sin(\alpha)$
- volume of parallelepiped: $|\vec{u} \cdot (\vec{v} \times \vec{w})|$
- parallel vectors $\vec{v} \times \vec{w} = 0$, orthogonal vectors: $\vec{v} \cdot \vec{w} = 0$
- scalar projection $\vec{v} \cdot \vec{w}/|\vec{w}|$
- vector projection $P_{\vec{w}}(\vec{v}) = (\vec{v} \cdot \vec{w})\vec{w}/|\vec{w}|^2$
- completion of square: $x^2 - 4x + y^2 = 1$ gives $(x-2)^2 + y^2 = 5$
- sphere: $(x-1)^2 + (y-b)^2 + (z-c)^2 = \rho^2$

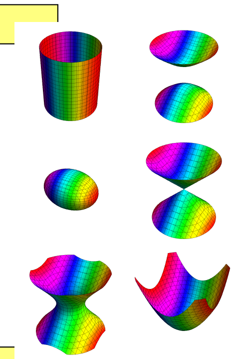
Lines, Planes, Functions





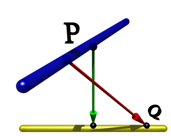
- parametric equation for plane $\vec{x} = \vec{x}_0 + t\vec{v} + s\vec{w}$
- plane $ax + by + cz = d$
- parametric equation for line $\vec{x} = \vec{x}_0 + t\vec{v}$
- symmetric equation of line $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$
- graph $G = \{(x, y, f(x, y)) \mid (x, y) \text{ in the domain of } f\}$
- plane $ax + by + cz = d$ has normal vector $\vec{n} = [a, b, c]$
- plane through A, B, C: us $[a, b, c] = \vec{AB} \times \vec{CB}$ and plug in point

Level surfaces

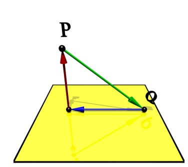


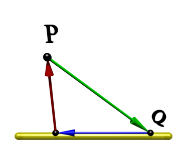
- intercepts: intersections of a surface with coordinate axis
- traces: intersections of a surface with coordinate planes
- generalized traces: intersections with $\{x = c\}, \{y = c\}$ or $\{z = c\}$
- level surface $g(x, y, z) = c$. Example: graph $g(x, y, z) = z - f(x, y)$
- linear equation like $2x + 3y + 5z = 7$ defines plane
- quadric: ellipsoid,paraboloid,hyperboloid,cylinder,cone

Distance formulas

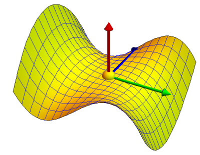


- point-point: $d(P, Q) = |\vec{PQ}| = \sqrt{(P_1 - Q_1)^2 + (P_2 - Q_2)^2 + (P_3 - Q_3)^2}$
- point-plane: $d(P, \Sigma) = |(\vec{PQ}) \cdot \vec{n}|/|\vec{n}|$
- point-line: $d(P, L) = |(\vec{PQ}) \times \vec{u}|/|\vec{u}|$
- line-line: $d(L, M) = |(\vec{PQ}) \cdot (\vec{u} \times \vec{v})|/|\vec{u} \times \vec{v}|$
- parallel lines: L, M: distance point $d(P, M)$, where P is in L.
- parallel planes: $d(\Pi, \Sigma) = d(P, \Sigma)$, where $P \in \Pi$ is point.

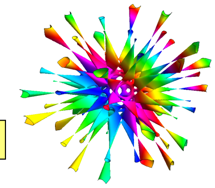




Functions

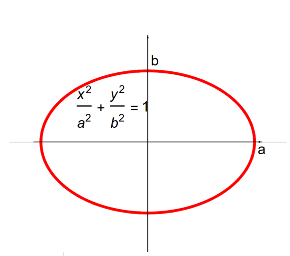


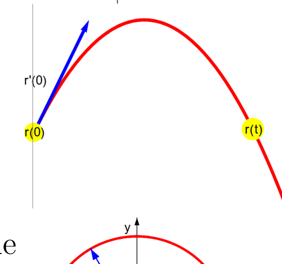
- graph: $z = f(x,y)$ is a surface in $\mathbb{R}^3$.
- contour set: $f(x,y) = c$ is usually a curve in $\mathbb{R}^2$
- contour map: draw all contour sets $\{f(x,y) = c\}$ for various c.
- contour surface: $f(x,y,z) = c$ is usually a surface in space
- space contour map: draw contour sets $\{f(x,y,c) = c\}$ for various c.



Curves

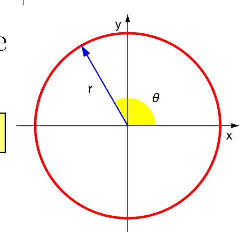
- plane curve $\vec{r}(t) = [x(t), y(t)]$
- space curve $\vec{r}(t) = [x(t), y(t), z(t)]$
- circle: $x^2 + y^2 = r^2$, $\vec{r}(t) = [r\cos t, r\sin t]$.
- ellipse: $(x - x_0)^2/a^2 + (y - y_0)^2/b^2 = 1$, $\vec{r}(t) = [x_0 + a\cos t, y_0 + b\sin t]$
- velocity $\vec{r}\,'(t)$, acceleration $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$ speed
- unit tangent vector $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$
- integration: get $\vec{r}(t)$ from $\vec{r}\,'(t)$ and $\vec{r}(0)$ by integration.
- integration: get $\vec{r}(t)$ from acceleration $\vec{r}\,''(t)$ as well as $\vec{r}\,'(0)$ and $\vec{r}(0)$.
- velocity $\vec{r}\,'(t)$ tangent to curve at $\vec{r}(t)$.
- polar curve $\vec{r}(t) = [f(t)\cos(t), f(t)\sin(t)]$ to polar graph $r = f(\theta)$.
- arc length $\int_a^b |\vec{r}\,'(t)|\, dt$ of parametrized curve. Param. independent!
- normal vector $\vec{N}(t) = \vec{T}'(t)/|\vec{T}'(t)|$ perpendicular to $\vec{T}(t)$.
- bi-normal $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$ perpendicular to $\vec{T}$ and $\vec{N}$.
- curvature $\kappa(t) = |\vec{T}\,'(t)|/|\vec{r}\,'(t)| = |\vec{r}\,'(t) \times \vec{r}\,''(t)|/|\vec{r}\,'(t)|^3$. Parameter inde

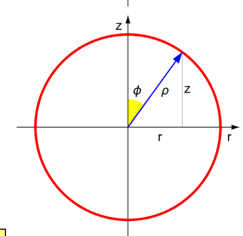




Coordinates

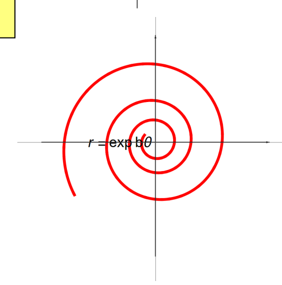
- Cartesian coordinates (x, y, z)
- polar coordinates $(x, y) = (r\cos(\theta), r\sin(\theta))$, $r \geq 0$
- cylindrical coordinates $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$, $r \geq 0$
- spherical coordinates $(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$
- radius: $r = x^2 + y^2$ and spherical radius $\rho = x^2 + y^2 + z^2$.
- polar radius: important relation $r = \rho\sin(\phi)$

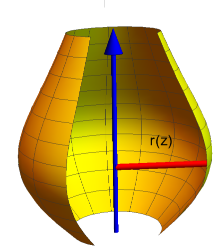




Surfaces

- $\vec{r}(u, v) = [x(u, v), y(u, v), z(u, v)]$ parametrization
- $\vec{r}(t) = \vec{r}(u_0, t)$, $\vec{r}(t) = \vec{r}(t, v_0)$ grid curves on surface
- $g(r, \theta) = 0$ polar curve, especially $r = f(\theta)$, polar graph
- $r = f(z, \theta)$ cylindrical surface, $r = r(z)$ surface of revolution
- $g(\rho, \theta, \phi) = 0$ spherical surface: example $\rho = 1$ sphere
- plane: $ax + by + cz = d$, $\vec{r}(s, t) = \vec{r}_0 + s\vec{v} + t\vec{w}$, $[a, b, c] = \vec{v} \times \vec{w}$
- surface of revolution: $x^2 + y^2 = r(z)^2$, $\vec{r}(\theta, z) = [r(z)\cos(\theta), r(z)\sin(\theta), z]$
- graph: $g(x, y, z) = z - f(x, y) = 0$, $\vec{r}(x, y) = [x, y, f(x, y)]$
- rotated graph $g(x, y, z) = y - f(x, z) = 0$, $\vec{r}(x, z) = [x, f(x, z), z]$
- ellipsoid: $\vec{r}(\theta, \phi) = [a\cos\theta\sin\phi, b\sin\theta\sin\phi, c\cos\phi]$
- unit sphere: $x^2 + y^2 + z^2 = 1$, $\vec{r}(u, v) = [\cos u\sin v, \sin u\sin v, \cos v]$





Oliver Knill, Math S 21a, last update: July 7, 2024