"I affirm my awareness of the standards of the Harvard College Honor Code."

Name:

- Start by writing your name in the above box.

- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader can not be given credit.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
Total:		120

Problem 1) (20 points) No justifications are needed.

1) T **F** The plane $x = 3$ does intersect the yz-plane.

Solution:

The xy plane is $x = 0$.

2) T **F** The curve $\vec{r}(t) = [1 + 2t, t, 1 + t]$ intersects the z-axis in a point.

Solution:

If $y = 0$, then x is not zero.

3) T **F** The Cauchy-Schwartz inequality states $|\vec{v} \cdot \vec{w}| \leq |\vec{v}|$ for any two vectors $\vec{v}, \vec{w}$.

Solution:

We also have to divide by the length of the vector $\vec{v}$.

4) T **F** The curvature of a circle $\vec{r}(t) = [\cos(2t), 0, \sin(2t)]$ is equal to $1/2$ everywhere.

Solution:

This is a unit circle, the curvature is 1.

5) **T** F The surface $y^2 - x + y^2 = 2$ is an elliptic paraboloid.

Solution:

It is an elliptic paraboloid.

6) T **F** The angle between the vectors $\vec{0A}$ and $\vec{0B}$ is positive if the distance between the points A and B is positive.

Solution:

The angle can become zero.

7) **T** F   Let $\vec{j} = [0, 1, 0]$. There is a vector $\vec{v}$ for which the vector projection of $\vec{v}$ onto $\vec{j}$ is equal to $-\vec{j}$.

Solution:

Take $\vec{v} = -\vec{j}$.

8) T **F**   Two particles with path $\vec{r}_1(t) = [0, t, -t]$ and $\vec{r}_2(t) = [1 - t, t - 1, 0]$ do collide.

Solution:

They do intersect as curves but not collide.

9) **T** F   In spherical coordinates the surface $\rho^2 \sin^2(\phi) - \rho^2 \cos^2(\phi) = 1$ is a one-sheeted hyperboloid.

Solution:

It is $x^2 + y^2 - z^2 = 1$.

10) **T** F   If $|\vec{u} \times \vec{v}| = 1$, for unit vectors $\vec{u}, \vec{v}$, then $\vec{u}, \vec{v}$ are orthogonal.

Solution:

We must have $\sin(\alpha) = 1$.

11) **T** F   The curve $r^2 \cos^2(\theta) - r^2 \sin^2(\theta) = 1$ in polar coordinates is a hyperbola.

Solution:

Yes, $x^2 - y^2 = 1$.

12) **[T]** [F]   If the arc length of a curve connecting A with B is 0, then $A = B$.

Solution:

The arc length is larger or equal than the distance.

13) **[T]** [F]   The surface parametrized as $\vec{r}(y, z) = [y, z, y^2 - z^2]$ is a hyperbolic paraboloid.

Solution:

Yes, it reads $z = x^2 + y^2$.

14) [T] **[F]**   The velocity vector and the acceleration are always either parallel or perpendicular.

Solution:

These are extreme cases but it is not true in general

15) **[T]** [F]   It is possible that a plane and a one-sheeted hyperboloid intersects in two crossing lines.

Solution:

See homework

16) **[T]** [F]   The function $f(x, y) = \log(x^2 + y^2)$ contains as domain all points except the origin $(0, 0)$.

Solution:

Yes, the log

17) **[T]** [F]   The normal vector $\vec{N}$ and the unit tangent vector $\vec{T}$ are perpendicular if $\vec{T}, \vec{T}\,'$ are both not zero vectors.

Solution:

We have proven this

18) **T** F   The distance between two non-parallel lines in three dimensional space can be zero.

Solution:

Yes, If they cross.

19) **T** F   If $\vec{i}, \vec{j}, \vec{k}$ denote the unit vectors in the x, y and z axis, then $\vec{i} \cdot (\vec{j} \times \vec{k}) = 1$.

Solution:

Yes it is the volume.

20) **T** F   For any two lines L, M, there are points P on L and Q on M such that $d(P, Q) = 2d(L, M)$, where $d(L, M)$ is the distance between the lines.

Solution:

By the intermediate value theorem

Problem 2) (10 points) No justifications are needed in this problem.
In each sub-problem, each of the numbers 0,1,2,3 each occur exactly once.

a) (2 points) Match the surfaces $g(x, y, z) = c$. Enter 0 if there is no match.
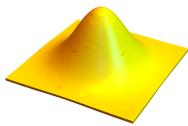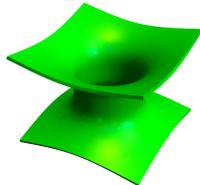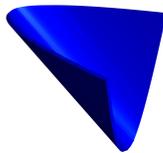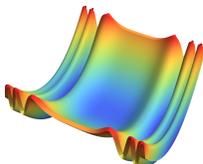
1   2   3

Function $g(x, y, z) =$	0,1,2, or 3
$x - y^2 + z = 0$	
$x^2 + y^2 - z^4 = 1$	
$z - e^{-x^2 - y^2} = 0$	
$y^2 + z^3 = 1$	

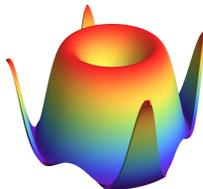b) (2 points) Match the graphs of the functions $f(x, y)$. Enter 0 if there is no match.
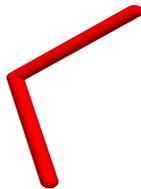
1   2   3

Function $f(x, y) =$	0,1,2, or 3
$\sqrt{|1 + x^2 - y^2|}$	
$\sin(x^2 + y^2)$	
$||x - y| - |x + y||$	
$y^2 \sin(x^4)$	

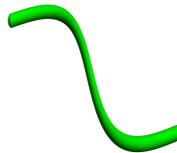c) (2 points) Match the space curves with the parametrizations. Enter 0 if there is no match.
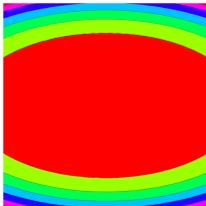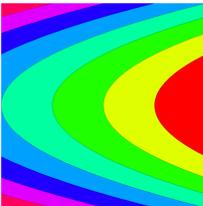
1   2   3

Parametrization $\vec{r}(t) =$	0,1,2, or 3
$[|t|, t, t]$	
$[\cos(2t), 0, \cos(2t)]$	
$[0, \cos(2t), \sin(2t)]$	
$[t, \sin(t), 0]$	

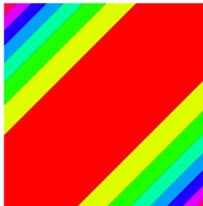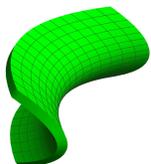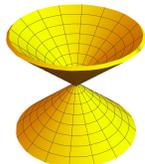d) (2 points) Match the functions g with contour plots in the xy-plane. Enter 0 if there is no match.

1   2   3

Function $g(x, y) =$	0,1,2, or 3
$\cos(2x) + \sin(2y)$	
$(x - y)^2$	
$y^2 - x$	
$(2x^2 + 7y^2)^2$	

e) (2 points) Match the parametrized surfaces. Enter 0 if there is no match.

1   2   3

Parametrization $\vec{r}(u, v) =$	0-3
$[u, u^2 + v^2, v]$	
$[\sin(v)\cos(u), \sin(v)\sin(u), \cos(v)]$	
$[u^2 - v^2, u, v]$	
$[u\cos(v), u\sin(v), u]$	

Problem 3) (10 points)

Let $\vec{v} = [2, 2, 1]$ and $\vec{w} = [1, 1, 1]$ and assume θ is the angle between $\vec{v}$ and $\vec{w}$.

a) (5 points) Compute $\vec{n} = \vec{v} \times \vec{w}$ and $a = \vec{v} \cdot \vec{w}$.

$\vec{n} =$

$a =$

b) (5 points) Compute $\sin^2(\theta)$ by using the vector $\vec{n}$ and $\cos^2(\theta)$ by using the scalar a and check that $\sin^2(\theta) + \cos^2(\theta)$ is equal to 1.

$$\sin^2(\theta) = \boxed{} \qquad \cos^2(\theta) = \boxed{}$$

Check that the sum is 1:

Problem 4) (10 points)

a) (5 points) Parametrize the plane $4x + 4y + 2z = 0$ using parameters s, t:

$$\vec{r}(s, t) = \boxed{}$$

b) (5 points) Find the distance between $4x + 4y + 2z = 0$ and the point $(2, 2, 2)$.

distance =

<div style="border:1px solid black; height:240px;"></div>

Problem 5) (10 points)

a) (5 points) Find a parametrization $\vec{r}(t)$ of the intersection of the planes

$$x + y + z = 1, \qquad\qquad 2x - y + 2z = 2 .$$

$\vec{r}(t) =$

<div style="border:1px solid black; height:180px;"></div>

b) (5 points) Find the distance between that line computed in a) and $P = (1, 1, 1)$.

distance =

Problem 6) (10 points)

a) (5 points) Find the arc length of the path

$$\vec{r}(t) = [\frac{3t^2}{2}, \frac{4t^2}{2}, \frac{5t^3}{3}]$$

with $0 \leq t \leq 1$.

Length =

b) (5 points) Find the curvature of $\vec{r}(t)$ at $t = 1$ using the vectors $\vec{v} = \vec{r}\,'(1), \vec{w} = \vec{r}\,''(1)$.

$$\kappa(\vec{r}(1))) = \boxed{}$$

Problem 7) (10 points)

a) (5 points) Given the **jerk**
$$\vec{r}'''(t) = [0, 0, -12]$$
with $\vec{r}(0) = [0, 0, 3], \vec{r}'(0) = [4, 0, 0], \vec{r}''(0) = [0, 2, 0]$, find $\vec{r}(t)$ and especially $\vec{r}(10)$.

$$\vec{r}(10) = \boxed{}$$

b) (5 points) Compute the unit tangent vector $\vec{T}$ of the TNB-frame to the curve $\vec{r}(t)$ at $t = 0$.

11

$$\vec{T}(0) = \boxed{}$$

Problem 8) (10 points)

We experiment with a **paper air plane**. The wing tips are $C = (0, 3, 4)$ and $D = (0, -3, 4)$ the front is $A = (3, 0, 0)$ the back is $B = (-3, 0, 0)$.

a) (5 points) The sum of the areas of the triangles ABC and ABD is the total wing area. Find the wing area.

$$\text{Area} = \boxed{}$$

b) (5 points) The volume of the tetrahedron with vertices A, B, C, D is known to be 1/6th of the volume of the parallelepiped spanned by AB, AC, AD. Find the volume of the tetrahedron.

$$\text{Volume} = \boxed{}$$

Problem 9) (10 points) No justifications are needed.

a) (2 points) Parametrize the surface $x^2/9 + y^2/4 + z^2/4 = 1$.

$$\vec{r}(\theta, \phi) = [, ,]$$

b) (2 points) Parametrize the surface $x^2 = y^2 + z^2$ using an angle θ in the yz-plane.

$$\vec{r}(\theta, x) = [, ,]$$

c) (2 points) Parametrize the surface $y = x^4 - 6x^2z^2 + z^4$.

$$\vec{r}(x, z) = [, ,]$$

d) (2 points) Parametrize the surface $x^2 + z^2 = 4$ using an angle θ in the xz-plane.

$$\vec{r}(\theta, y) = [, ,]$$

e) (2 points) Parametrize the surface $(x - 7)^2 + z^2 - y^2 = 8$.

$$\vec{r}(\theta, y) = [, ,]$$

The following pictures are here just to be admired and can also be ignored.
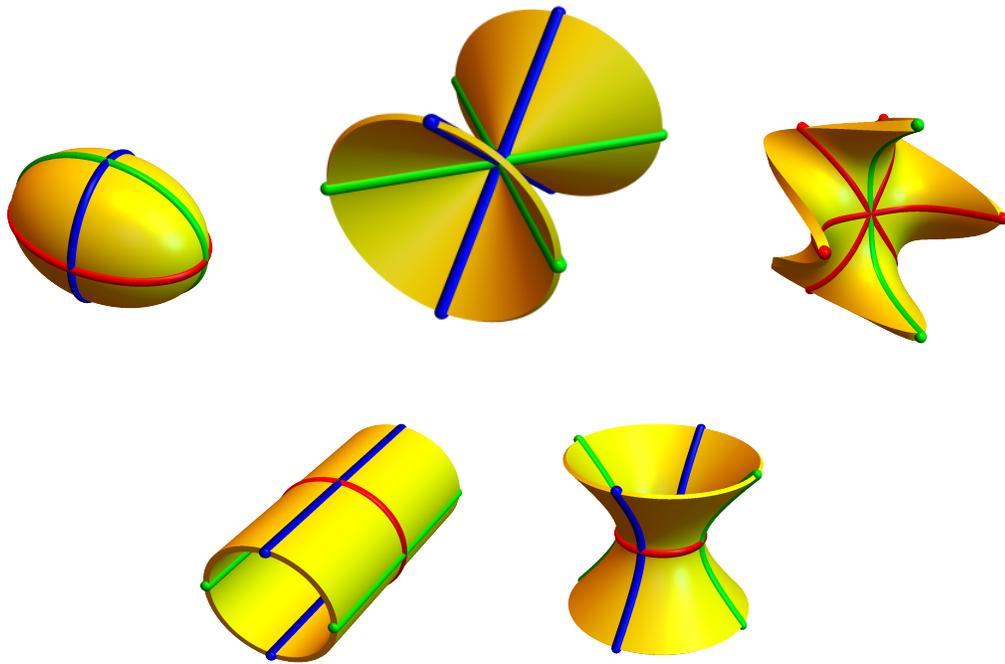
Solution:

a) $[3\sin(\phi)\cos(\theta), 2\sin(\phi)\sin(\theta), 2\cos(\phi)]$.
b) $[x, x\cos(\theta), x\sin(\theta)]$.
c) $[x, x^4 - 6x^2z^2 + z^4, z]$.
d) $[2\cos(\theta), y, 2\sin(\theta)]$.
e) $[7 + \sqrt{8 + y^2}\cos(\theta), y, \sqrt{8 + y^2}\sin(\theta)]$.