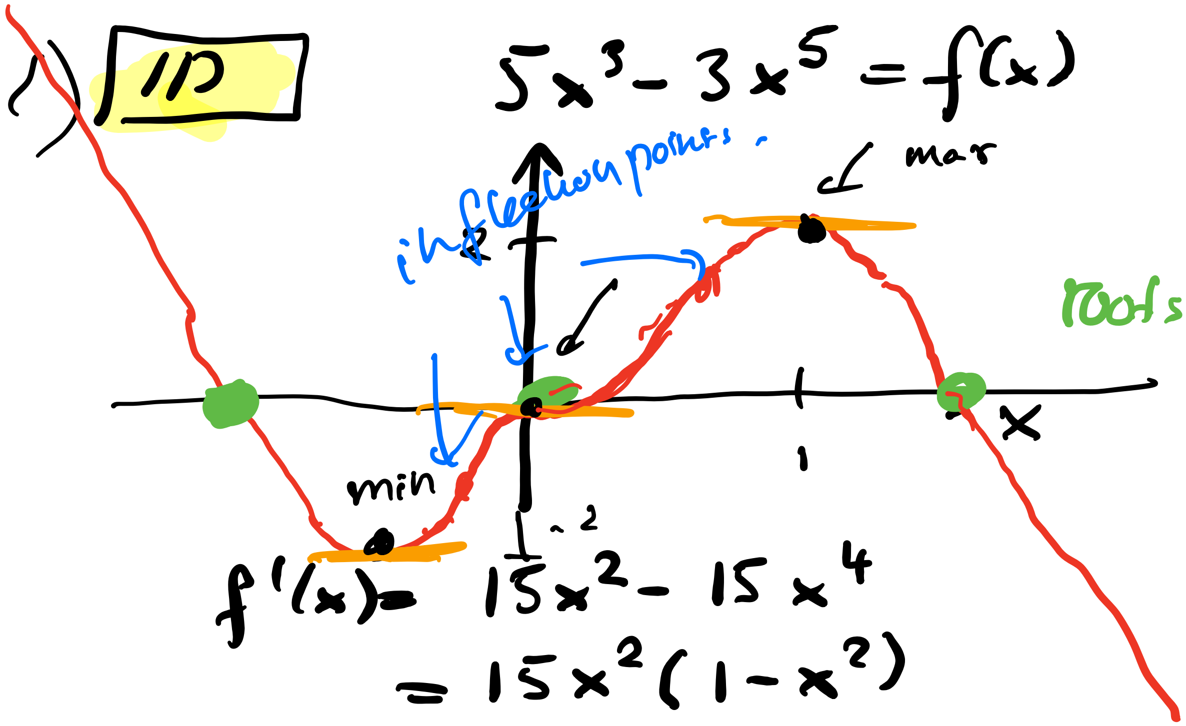
Unit 13

1) $\boxed{1D}$

$$5x^3 - 3x^5 = f(x)$$



$$f'(x) = 15x^2 - 15x^4$$
$$= 15x^2(1-x^2)$$

0, 1, -1 are critical point.

$$f''(x) = 30x - 60x^3$$

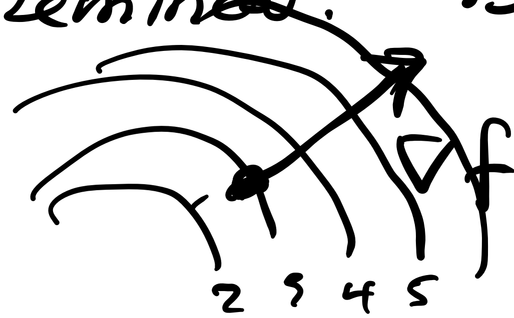
at 1 : $f''(1) < 0$
at -1 : $f''(-1) > 0$
at 0 : $f''(0) = 0$

We generalize the tools today for higher dimensions.

2) Fermat principle

$f(x,y)$

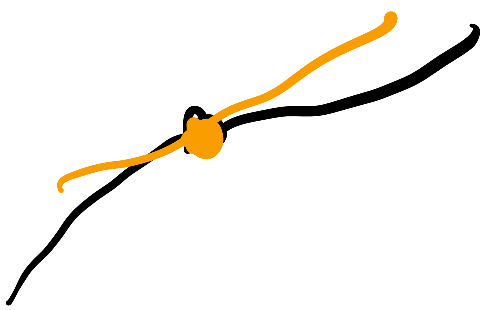
Reminder: $D_{\frac{\nabla f}{|\nabla f|}} f = |\nabla f|$



Steepest ascent is the direction of the gradient

One of paradigms of machine learning

Fermat:



$\boxed{\text{If } |\nabla f| \neq 0 \text{ then you can not be on a max or min.}}$

Turned around.

$\boxed{\text{If } (x_0,y_0) \text{ is a max, then } \nabla f(x_0,y_0) = [0,0]}$

we call a point, where
$\nabla f(x_0, y_0) = [0, 0]$ a
critical point

3) 2. Derivative theorem

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

Hessian matrix
organizes all the four second
derivatives.

$$D = \det H$$
$$= f_{xx} f_{yy} - (f_{xy})^2$$

is called the
discriminant

Examples!

a) $f(x,y) = x^2 + y^2$

min

$$\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$H = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$f_{xx} > 2$, $D = 4$

b) $f(x,y) = x^2 - y^2$

Sad

$$\nabla f = \begin{bmatrix} 2x \\ -2y \end{bmatrix}$$

$$H = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

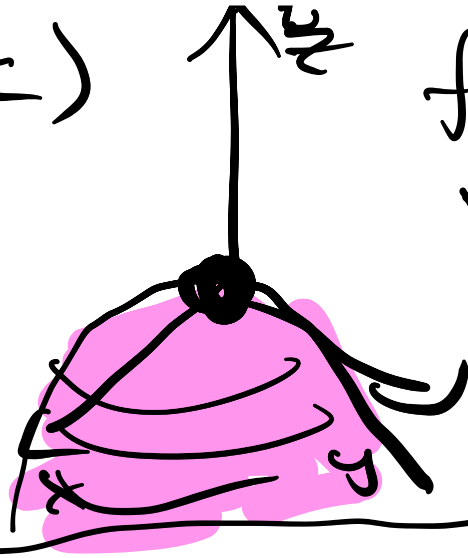
$D = -4$

Saddle

mountain pass.

neither max, nor min.

c)



$$f(x,y) = -x^2 - y^2$$

$$\nabla f = \begin{bmatrix} -2x \\ -2y \end{bmatrix}$$

$$H = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

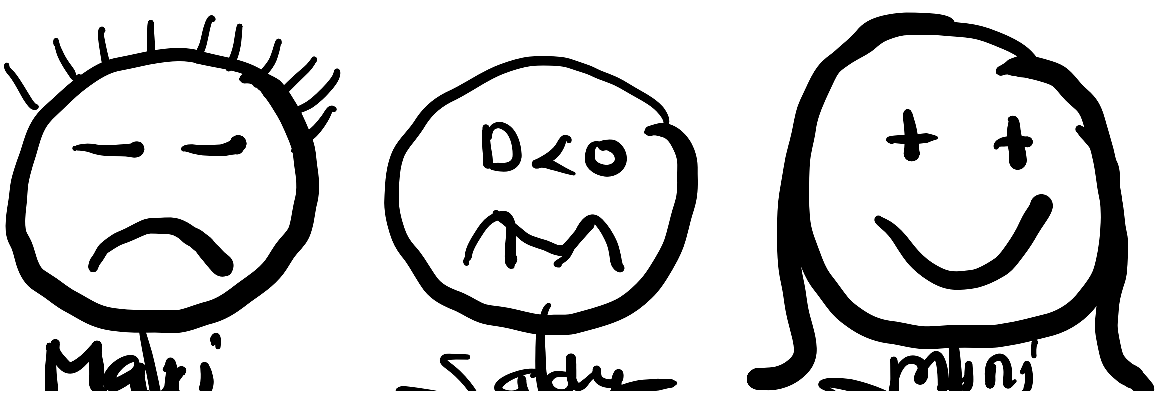
$$D = 4$$

Theorem: (x_0, y_0) is a critical point.

i) $D > 0$, $f_{xx} > 0 \Rightarrow$ **Min**

ii) $D > 0$, $f_{xx} < 0 \Rightarrow$ **Max**

iii) $D < 0 \Rightarrow$ **Sad**



4) Examples

ipad → zoom
connection on
strike.