Unit 4

4.1 Planes

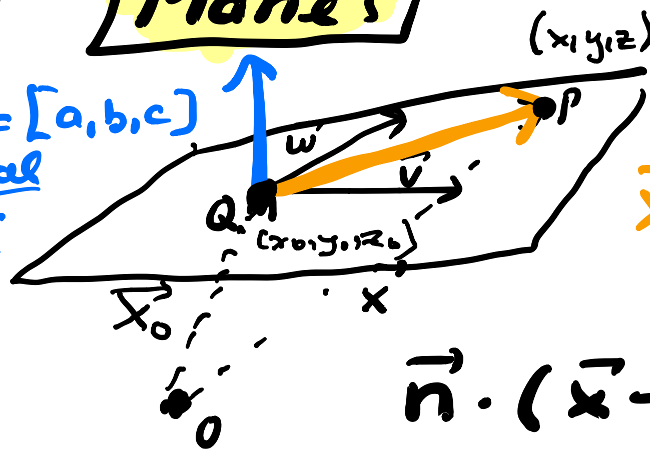
$\vec{x_0} = \vec{OQ}$

$= \vec{OP}$

$\vec{x} = [x, y, z]$

$\vec{n} = [a, b, c]$

$\vec{n} = [a, b, c]$ normal vector



$\vec{x} - \vec{x_0}$

$$\vec{n} \cdot (\vec{x} - \vec{x_0}) = 0$$

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{x_0} = d$$

implicit equation

$$\boxed{ax + by + cz = d}$$

equation of a plane.

$d = ax_0 + by_0 + cz_0$

obtaind by pluggin Q into the equation

$$\boxed{3x + 4y + 7z = 100}$$

is a plane we see the normal vector to this plane

$\vec{n} = [3, 4, 7]$ is normal.

mizaks

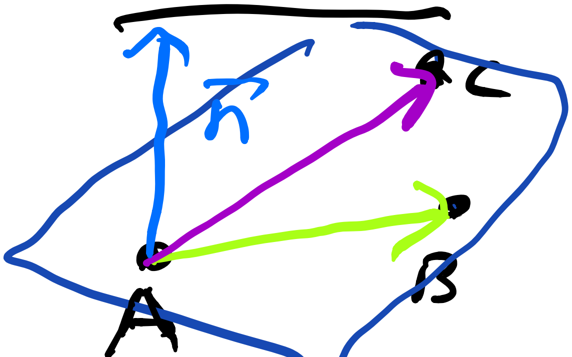
Parame

There is a second way to describe a plane: parametrization

$$\boxed{\vec{x} = \vec{x}_0 + s\vec{v} + t\vec{w}}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \vec{r}(t,s) = \begin{bmatrix}x_0\\y_0\\z_0\end{bmatrix} + s\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} + t\begin{bmatrix}w_1\\w_2\\w_3\end{bmatrix}$$

Problem: Find the plane through the points



$A = (3,1,2)$

$B = (2,4,6)$

$C = (1,1,1)$

$\vec{v} = \overrightarrow{AB} = [-1,3,4]$

$\vec{w} = \overrightarrow{AC} = [-2,0,-1]$

$[a,b,c] = \vec{n} = \vec{v} \times \vec{w} = [-3,-9,6]$

Equation of the plane is

$$\boxed{(-3)x - 9y + 6z = -6}$$

plug in any point

The parametrization is:

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} + s\begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}}$$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 - s - 2t \\ 1 + 3s + 0 \\ 2 + 4s - t \end{bmatrix}}$$

pick s, t and get a point $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

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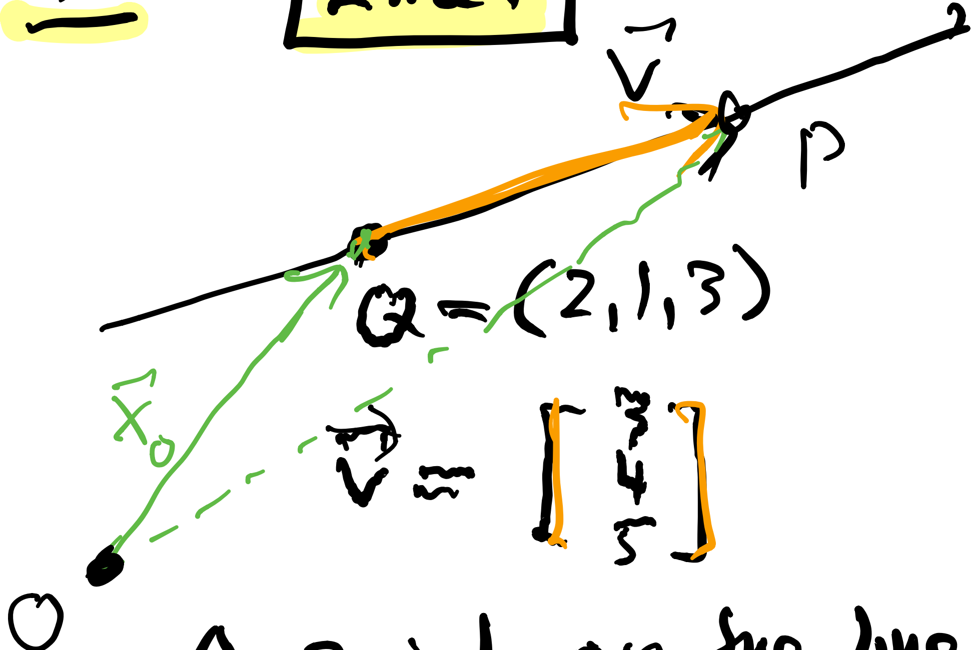
s, t are parameters.

The parametrization gives us from parameters s, t points on the plane.

4.2 Lines



$$\vec{v} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

A point on the line is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{OQ} + t \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\boxed{\vec{x} = \vec{x}_0 + t\vec{v}}$$

This is called a <u>parametrization of a line</u>.

If $t=0$, then $\vec{x} = \vec{OQ}$,

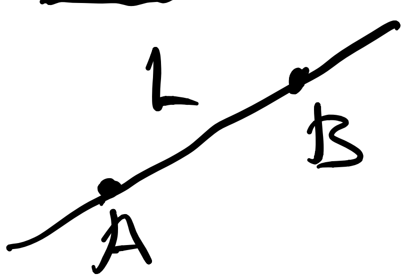
Think of t as a time parameter and of $\vec{v}$ as a velocity.

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \overset{\vec{v}}{\begin{bmatrix}2\\1\\3\end{bmatrix}} + t\begin{bmatrix}3\\4\\5\end{bmatrix}$$

$$= \begin{bmatrix}2+3t\\1+4t\\3+5t\end{bmatrix}$$

is a parametrization of the line.

Problem



Find the parametrization of the line through
$A = (1,2,3)$
$B = (7,-1,5)$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix} + t\begin{bmatrix}6\\-3\\2\end{bmatrix}$$

$$= \begin{bmatrix}1+6t\\2-3t\\3+2t\end{bmatrix}$$

at $t=0$ we are at A

A time $t=1$ we are the point B.

Think of t as time!

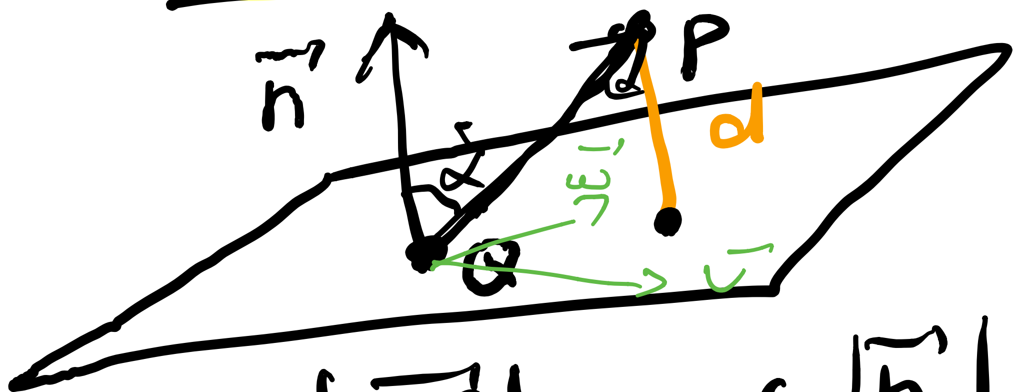
4.3 Distance Point Line

Distance $(P, L) =$

$$= |\overrightarrow{QP}| \cdot \sin\alpha \, \frac{|\vec{v}|}{|\vec{v}|} \quad = 1$$

$$= \boxed{\frac{|\vec{v} \times \overrightarrow{QP}|}{|\vec{v}|}} = \frac{\text{Area}}{\text{Base len}} = \text{height}.$$

4.4 Distance Point - Plane



$$d = |\overline{QP}| \cdot \cos\alpha \, \frac{|\vec{n}|}{|\vec{n}|}$$

$$= \boxed{\frac{|\overline{QP} \cdot \vec{n}|}{|\vec{n}|}}$$

if $\vec{n} = \vec{v} \times \vec{w}$, then

$$\frac{|\overline{QP} \cdot (\vec{v} \times \vec{w})|}{|\vec{v} \times \vec{w}|} = \frac{\text{Volume}}{\text{Area}} = \text{height}$$

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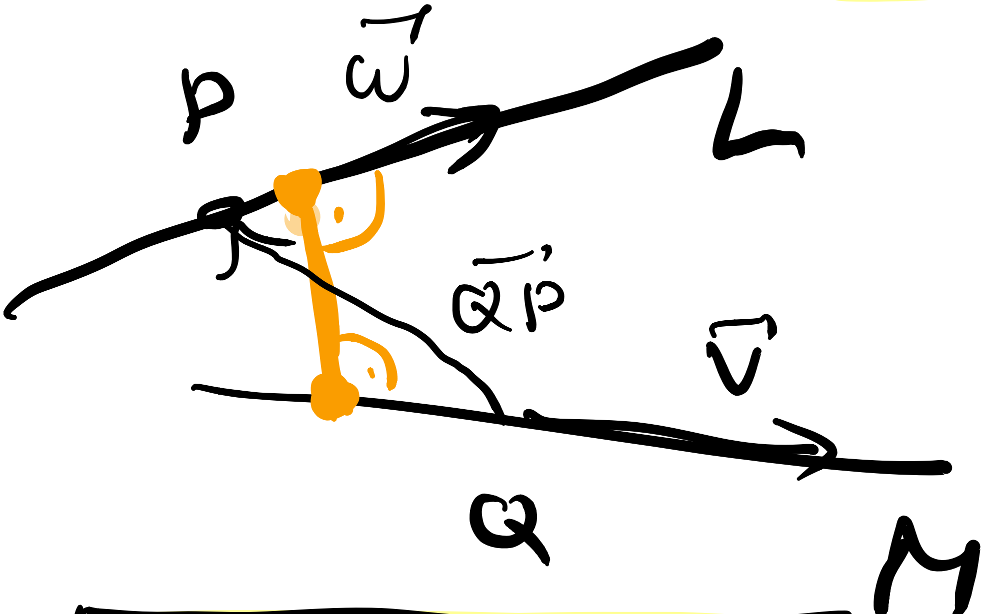
distance is also the length of the projection of $\overrightarrow{QP}$ onto $\vec{n}$

4.5 Distane Line - Line



$$\boxed{d = \frac{|\vec{QP} \cdot \vec{n}|}{|\vec{n}|}}$$

Projection of $\vec{QP}$ on $\vec{n} = \vec{v} \times \vec{w}$

$$d = \frac{|\vec{QP} \cdot (\vec{v} \times \vec{w})|}{|\vec{v} \times \vec{w}|}$$

This is again a volume divided by the base area.