Unit 1

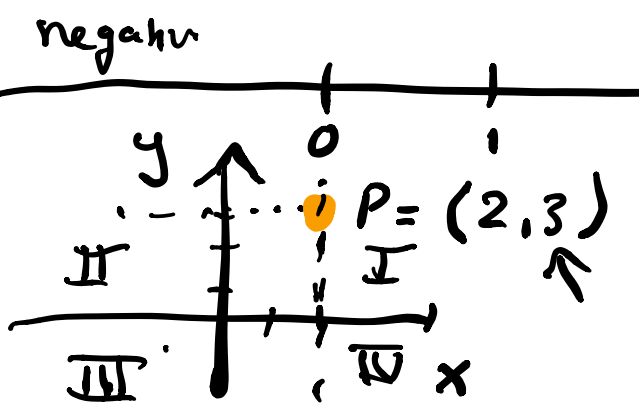
Geometry of space

① Coordinates

1637 La Geometry

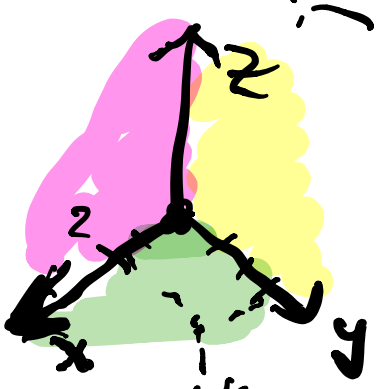
Descartes

1D



2D

3D

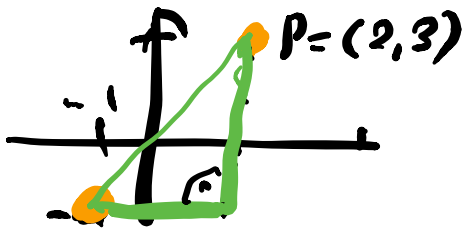


- axis
- coordinate planes xy
  yz plane

octants

4D . . . .

② Distances



$$d(P,Q) = 5$$

$$\sqrt{(2-(-1))^2+(3-(-1))^2}$$

motivated by Pythagoras theorem.
but it is a def

$$d((x,y),(a,b)) = \sqrt{(x-a)^2+(y-b)^2}$$

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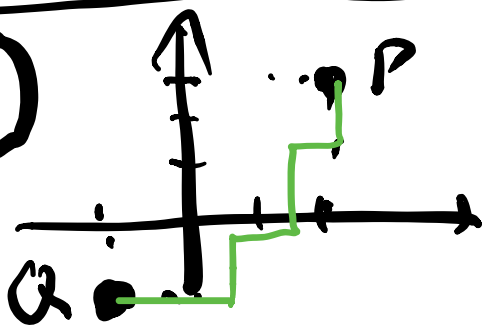
④



Taxi metric

$$|x-a|+|y-b|$$

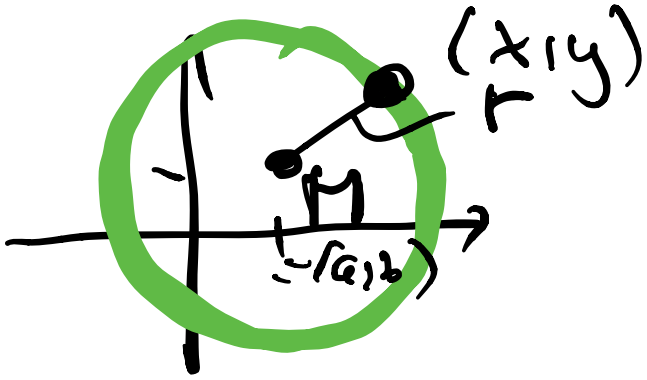
Einstein Relativity.

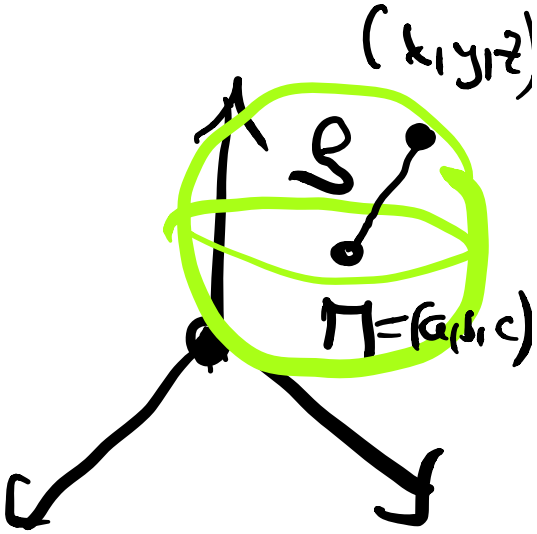
$$d((x,y,z),(a,b,c))$$

$$= \sqrt{(x-a)^2+(y-b)^2+(z-c)^2}$$

Important: distances are crucial.

③ Sphere





Circle equation

$\rho = r$

"greek r" called ρ

$$(x-a)^2+(y-b)^2=r^2$$

circle

$$(x-a)^2+(y-b)^2+(z-c)^2=\rho^2$$

sphere

Sphere geometry

Descartes

④ Completion of Square

Al Kwarizmi:

Start of algebra:

Q: What object is

$$x^2 - 4x + 4 + y^2 + 12y + 36 + z^2 = 10 + 4 + 36$$

$$(x-2)^2 + (y+6)^2 + z^2 = 50$$

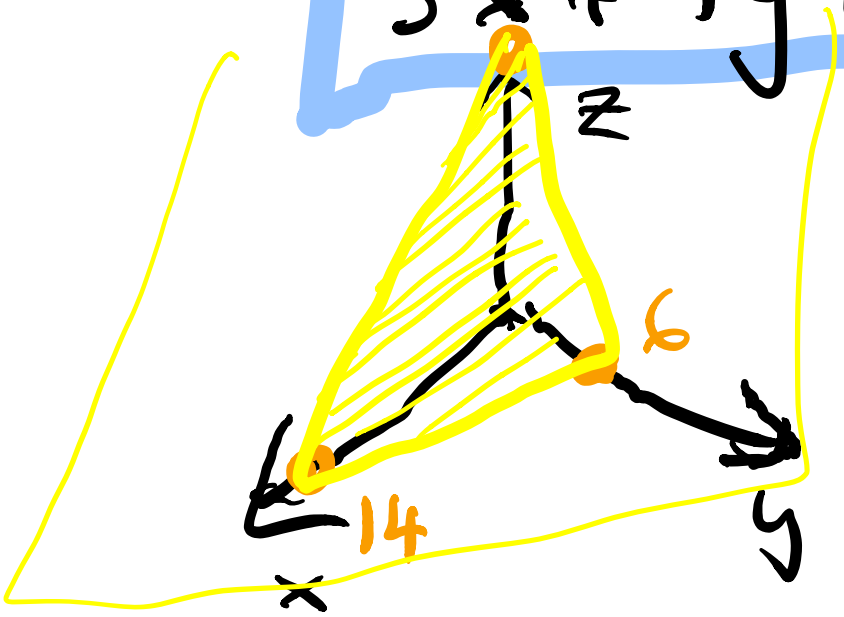
This is a sphere
centered at $H = (2, -6, 0)$
and radius $R = \sqrt{50} = r$

(5) **Objects**

a) What is

$$3x + 7y + z = 42$$

intercept

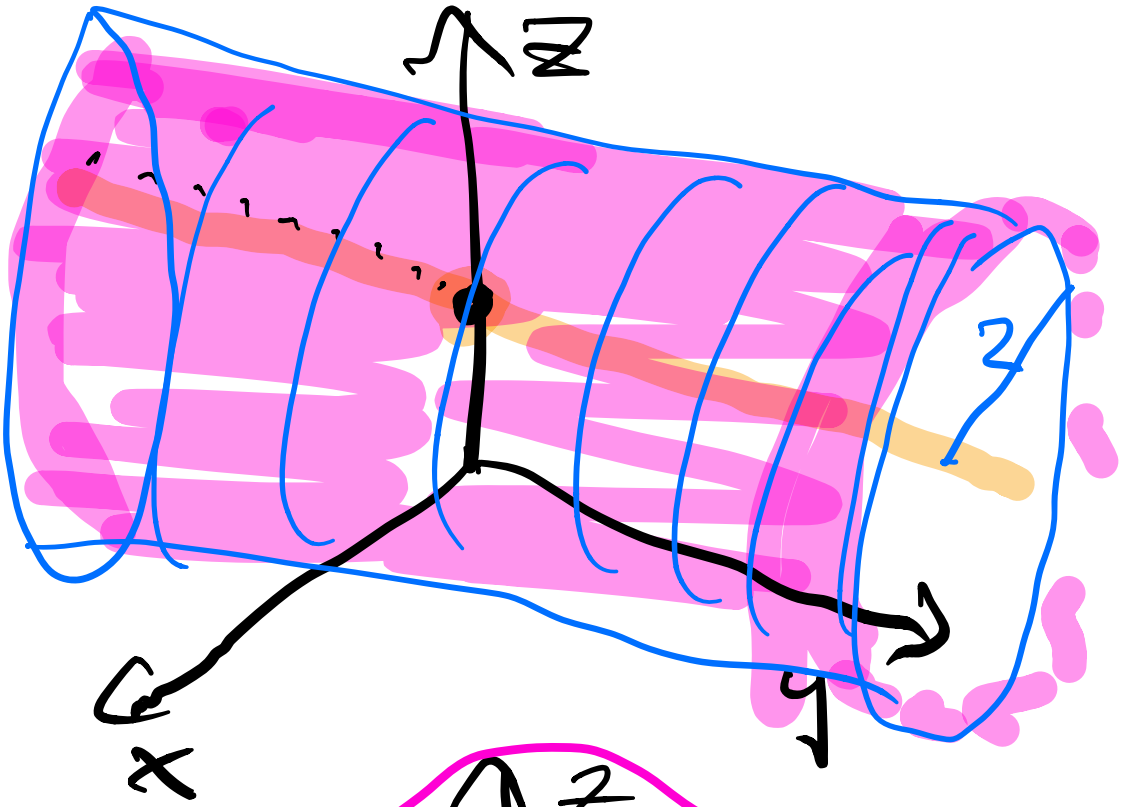


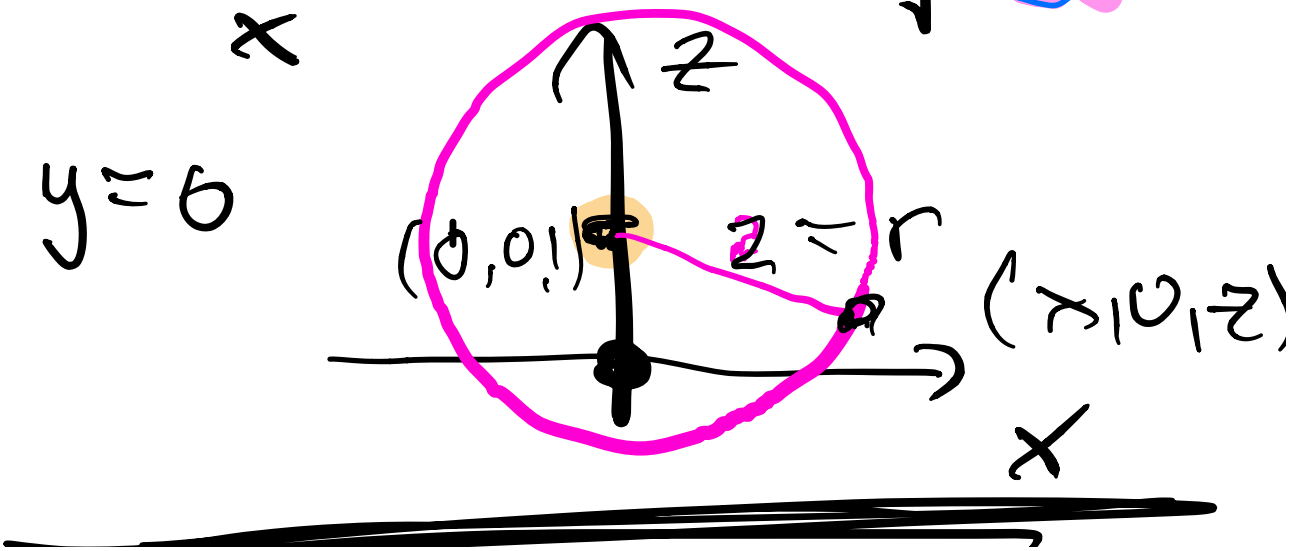
b) what is

$$x^2 + (z-1)^2 = 4$$

This is a cylinder:

For every y value, we have a circle.





2D analog

$x^2 + (y-1)^2 = 4 = r^2$

is a circle of radius 2 in the xy plan.

$(0, +1)$

So! $x^2 + y^2 = 1$

in $\mathbb{R}^2$ is a circle

but $x^2 + y^2 = 1$

in $\mathbb{R}^3$ is a

cylinder.