Second hourly: Checklist

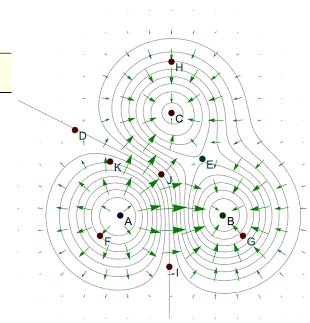
Partial Derivatives

- $f_x(x,y) = \frac{\partial}{\partial x} f(x,y)$ partial derivative
- $L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$ linear approximation
- $L(x,y)$ estimates $f(x,y)$ near $f(x_0,y_0)$. The result is $f(x_0,y_0) + a(x-x_0) + b(y-y_0)$
- tangent line: $ax + by = d$ with $a = f_x(x_0,y_0), b = f_y(x_0,y_0)$, $d = ax_0 + by_0$
- tangent plane: $ax + by + cz = d$ with $a = f_x, b = f_y, c = f_z$, $d = ax_0 + by_0 + cz_0$
- estimate $f(x,y,z)$ by $L(x,y,z)$ near (x_0,y_0,z_0)
- $f_{xy} = f_{yx}$ Clairaut's theorem, if f_{xy} and f_{yx} are continuous.
- $\vec{r}_u(u,v)$, $\vec{r}_v(u,v)$ tangent to surface parameterized by $\vec{r}(u,v)$

Partial Differential Equations

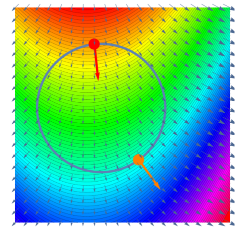
- $f_t = f_{xx}$ heat equation
- $f_{tt} - f_{xx} = 0$ wave equation
- $f_x - f_t = 0$ transport equation
- $f_{xx} + f_{yy} = 0$ Laplace equation
- $f_t + f f_x = f_{xx}$ Burgers equation
- $f_x^2 + f_y^2 = 1$ Eiconal equation
- $f_t = f - x f_x - x^2 f_{xx}$ Black Scholes

Gradient



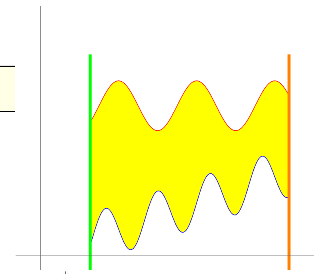
- $\nabla f(x,y) = [f_x, f_y]$, $\nabla f(x,y,z) = [f_x, f_y, f_z]$, gradient
- a vector of length 1 is a **direction** also called unit vector
- $D_{\vec{v}} f = \nabla f \cdot \vec{v}$ directional derivative, $\vec{v}$ is direction
- $\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$ chain rule
- $\nabla f(x_0,y_0)$ is orthogonal to the level curve $f(x,y) = c$ containing (x_0,y_0)
- $\nabla f(x_0,y_0,z_0)$ is orthogonal to the level surface $f(x,y,z) = c$ containing (x_0,y_0,z_0)
- $\frac{d}{dt} f(\vec{x} + t\vec{v}) = D_{\vec{v}} f$ by chain rule
- $(x-x_0) f_x(x_0,y_0) + (y-y_0) f_y(x_0,y_0) = 0$ tangent line
- $(x-x_0) f_x(x_0,y_0,z_0) + (y-y_0) f_y(x_0,y_0,z_0) + (z-z_0) f_z(x_0,y_0,z_0) = 0$ tangent plane
- directional derivative at (x_0,y_0) is maximal in the $\vec{v} = \nabla f(x_0,y_0)/|\nabla f(x_0,y_0)|$ direction
- $f(x,y)$ increases in the $\nabla f/|\nabla f|$ direction if $|nabla f| \neq 0$
- partial derivatives are special directional derivatives: $D_{\vec{i}} f = f_x$
- if $D_{\vec{v}} f(\vec{x}) = 0$ for all $\vec{v}$, then $\nabla f(\vec{x}) = \vec{0}$
- $f(x,y,z) = c$ defines $z = g(x,y)$, and $g_x(x,y) = -f_x(x,y,z)/f_z(x,y,z)$ implicit diff

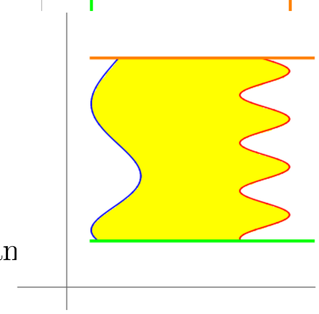
Extrema



- $\nabla f(x,y) = [0,0]$, critical point
- $D = f_{xx} f_{yy} - f_{xy}^2$ discriminant as used in second derivative test
- $f(x_0,y_0) \geq f(x,y)$ in a neighborhood of (x_0,y_0) local maximum
- $f(x_0,y_0) \leq f(x,y)$ in a neighborhood of (x_0,y_0) local minimum
- $\nabla f(x,y) = \lambda \nabla g(x,y), g(x,y) = c$, λ Lagrange equations
- $\nabla f(x,y,z) = \lambda \nabla g(x,y,z), g(x,y,z) = c$, λ Lagrange equations
- 2. deriv. test: $\nabla f = (0,0), D > 0, f_{xx} < 0$ **local max**
- 2. deriv. test: $\nabla f = (0,0), D > 0, f_{xx} > 0$ **local min**
- 2. deriv. test: $\nabla f = (0,0), D < 0$ **saddle point**
- $f(x_0,y_0) \geq f(x,y)$ everywhere, global maximum
- $f(x_0,y_0) \leq f(x,y)$ everywhere, global minimum

Double Integrals





- $\int\int_R f(x,y)\ dydx$ double integral
- $\int_a^b \int_c^d f(x,y)\ dydx$ integral over rectangle
- $\int_a^b \int_{c(x)}^{d(x)} f(x,y)\ dydx$ bottom-to-top region
- $\int_c^d \int_{a(y)}^{b(y)} f(x,y)\ dxdy$ left-to-right region
- $\int\int_R f(r,\theta) \boxed{r}\ dr d\theta$ polar coordinates
- $\int_a^b \int_c^d f(x,y)\ dydx = \int_c^d \int_a^b f(x,y)\ dxdy$ Fubini
- $\int\int_R \boxed{1}\ dxdy$ area of region R
- $\int\int_R f(x,y)\ dxdy$ signed volume of solid bound by graph of f and xy-plan

Surface Area

- $\int\int_R |\vec{r}_u \times \vec{r}_v|\ dudv$ surface area

General advise

- Draw the region when integrating in in higher dimensions.
- Consider other coordinate systems if the integral does not work.
- Consider changing the order of integration if the integral does not work.
- If an integral gets too complex, track back your steps to spot errors.
- For tangent planes, compute the gradient $[a,b,c]$ first then worry about the constant.
- When looking at relief problems, mind the gradient. It is the key!