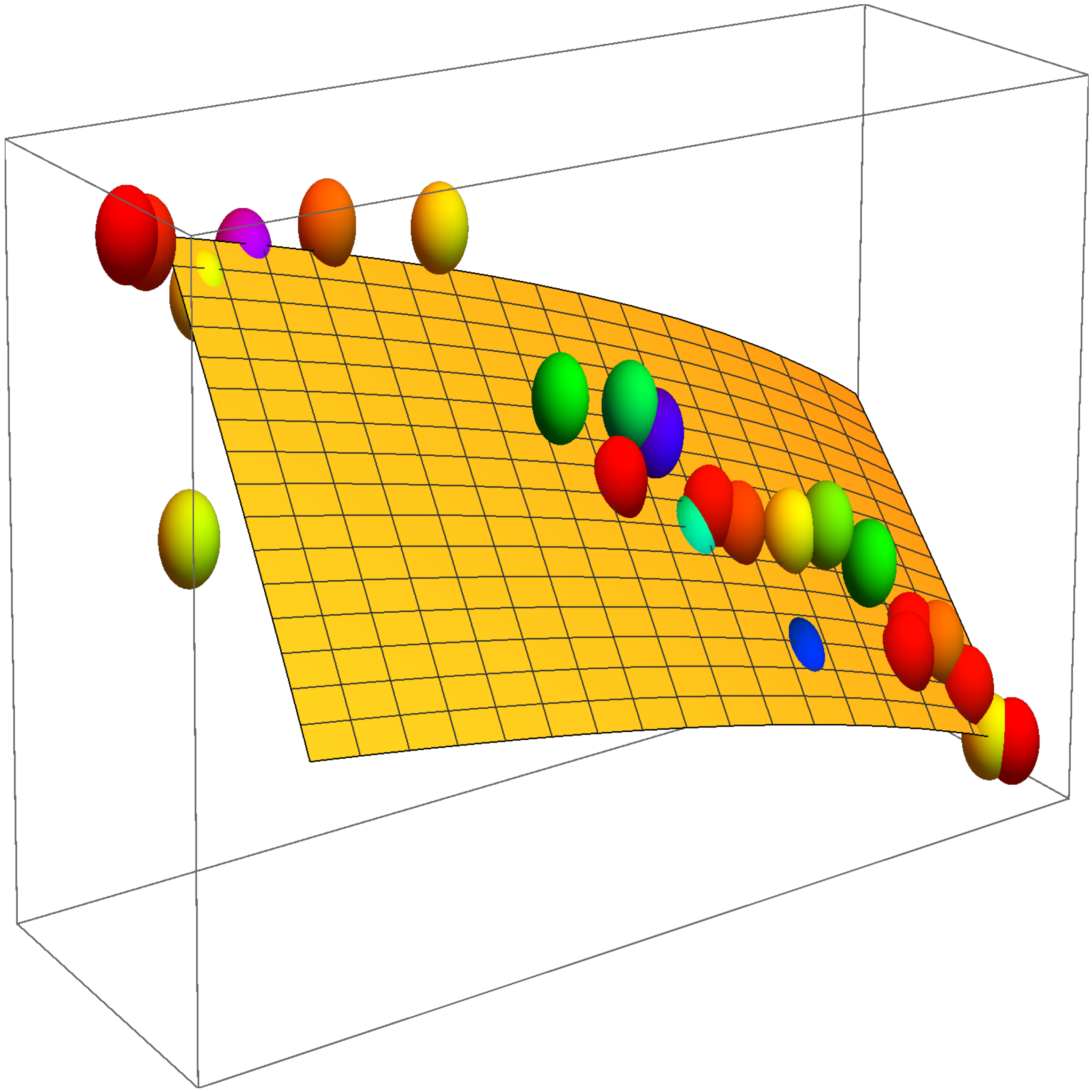
MULTIVARIABLE CALCULUS

MATH S-21A

Data illustration 3: Cobb Douglas

2.1. The mathematician and economist **Charles W. Cobb** at Amherst college and the economist and politician **Paul H. Douglas** who was also teaching at Amherst, found in 1928 empirically a formula $F(K, L) = L^\alpha K^\beta$ which fits the **total production** F of an economic system as a function of the **capital investment** K and the **labor** L. The two authors used logarithms variables and assumed linearity to find α, β. Below are the data normalized so that the date for year 1899 has the value 100.

$Year$	K	L	P
1899	100	100	100
1900	107	105	101
1901	114	110	112
1902	122	118	122
1903	131	123	124
1904	138	116	122
1905	149	125	143
1906	163	133	152
1907	176	138	151
1908	185	121	126
1909	198	140	155
1910	208	144	159
1911	216	145	153
1912	226	152	177
1913	236	154	184
1914	244	149	169
1915	266	154	189
1916	298	182	225
1917	335	196	227
1918	366	200	223
1919	387	193	218
1920	407	193	231
1921	417	147	179
1922	431	161	240



The graph of $F(L, K) = L^{3/4}K^{1/4}$ fits pretty well that data set.

Problem 1: Before we start with calculus, can you see in the data, which data point is an out-layer? You might see it by looking at differences of values from one year to the next, which is a time derivative derivative.

Problem 2: Assume that the labor and capital investment are bound by the additional constraint $G(L, K) = L^{3/4} + K^{1/4} = 50$. (This function G is unrelated to the function $F(L, K)$ as we are in a Lagrange problem.) Where is the production P maximal under this constraint? Plot the two functions $F(L, K)$ and $G(L, K)$.