7/6/2017 FIRST HOURLY Practice IV **Maths 21a, O.Knill, Summer 2017**

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Except for problems 1-2, give details.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

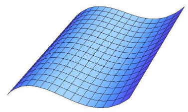
Problem 1) (20 points) No justifications are needed.

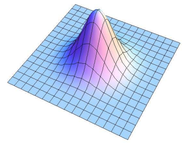
1)	T	F	$\langle 0,0,1\rangle \times (\langle 0,1,0\rangle \times \langle 0,1,0\rangle) = (\langle 0,0,1\rangle \times \langle 0,1,0\rangle) \times \langle 0,1,0\rangle$.
2)	T	F	The unit tangent vector $\vec{T}$ is perpendicular to the velocity vector $\vec{r}\,'(t)$.
3)	T	F	The point given in spherical coordinates as $(\rho, \phi, \theta) = (2, \pi/2, \pi/2)$ is on the y axes.
4)	T	F	On a slide shown in class, a T-shirt had the formula $(x-a)^2 + (y-b)^2 = r^2$ with subtitle *I feel fabulously hyperbolic today!*
5)	T	F	The curvature of a circle of radius 3 is 3.
6)	T	F	The triple scalar product of three vectors can not be negative because it computes volumes.
7)	T	F	If the dot product between two vectors is negative, then the two vectors form an obtuse angle.
8)	T	F	The equality $\lvert v\rvert^2 \lvert w\rvert^2 - (\vec{v} \cdot \vec{w})^2 = \lvert \vec{v} \times \vec{w}\rvert^2$ is called the Cauchy-Schwarz equality.
9)	T	F	The surface given in cylindrical coordinates as $z^2 + r^2 = 1$ is a sphere.
10)	T	F	The arc length of the curve $\langle \sin(t), \cos(t)\rangle$ from 0 to 1 is equal to 1.
11)	T	F	Three lines in space always intersect in at least one point.
12)	T	F	The curve $\vec{r}(t) = \langle \cos(t), \sin(t), t\rangle$ hits the plane $z = 0$ at a right angle.
13)	T	F	The lines $\vec{r}(t) = \langle t, -t, 2t\rangle$ and $\langle 5-t, 3+t, -2t$ are parallel.
14)	T	F	The parametrized curve $\langle 2\cos(t), 0, 3\sin(t)\rangle$ is an ellipse.
15)	T	F	If a cone is intersected with a plane, we always obtain an ellipse.
16)	T	F	The vector $\langle 3/5, 0, 4/5\rangle$ is a direction.
17)	T	F	Two vectors $\vec{v}$ and $\vec{w}$ are parallel or anti-parallel if $\vec{v} \times \vec{w} = \vec{0}$.
18)	T	F	$\vec{u} \times (\vec{v} \times \vec{u}) = 0$ for all vectors $\vec{u}, \vec{v}$.
19)	T	F	The plane parametrized by $\vec{r}(t,s) = \langle t, s, 1\rangle$ is the same than $z = 1$.
20)	T	F	If $f(x,y) = x^2 y^2$, then $f_{xyxy} = 4$.

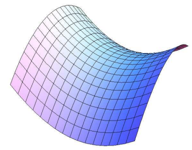
Total

Problem 2) (10 points) No justifications are needed in this problem.

a) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.

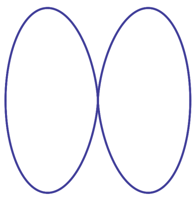

I

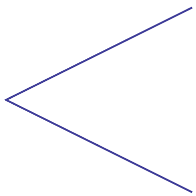

II


III

Function $f(x, y) =$	Enter O,I,II or III
$x^3 - y$	
$x^2 - y^2$	
x^4	
$\exp(-x^2 - y^2)$	
$\exp(-y^2)$	

b) (2 points) Match the plane curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.


I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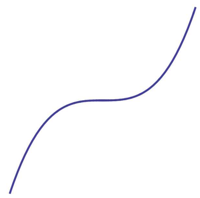
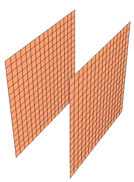
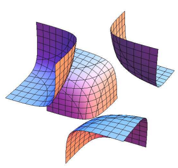

III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\vec{r}(t) = \cos^2(t)\langle\cos(t), \sin(t)\rangle$	
$\vec{r}(t) = \langle\cos^2(t), \sin^2(t)\rangle$	
$\vec{r}(t) = \langle t, t^3\rangle$	
$\vec{r}(t) = \langle\lvert t\rvert, t\rangle$	

c) (2 points) Match functions g with level surface $g(x, y, z) = 1$. Enter O, if no match.


I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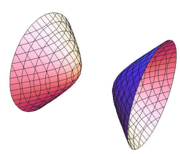
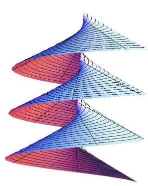
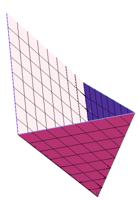

III

Function $g(x, y, z) = 1$	O, I,II,III
$x + y^2 + z^2 = 1$	
$xyz = 1$	
$x^2 = 1$	
$x^2 - y^2 - z^2 = 1$	

d) (2 points) Match the parametrization. Enter O, where no match.


I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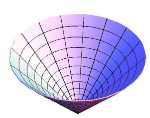
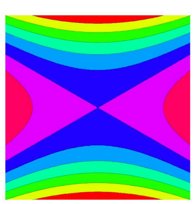

III

$\vec{r}(s, t)$	O,I,II,III
$\langle t, t^3 - s^3, s\rangle$	
$\langle s^2\cos(t), s^2\sin(t), s^2\rangle$	
$\langle s\cos(t), s\sin(t), t - s\rangle$	
$\langle\lvert t\rvert, \lvert s\rvert, \lvert t + s\rvert\rangle$	

e) (2 points) Match the contour maps. Enter O, where no match.


I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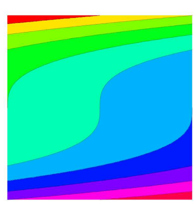
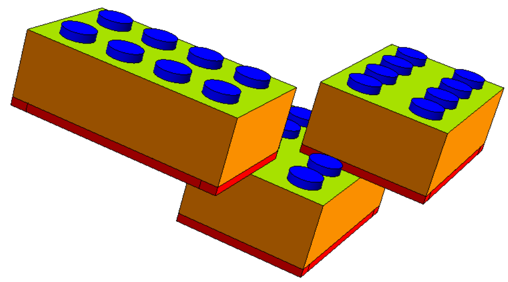

III

$f(x, y)$	O,I,II,III
$x - y^3$	
$x^2 + 3y^2$	
$-x^2 + 3y^2$	
$x - y^2$	

Problem 3) (10 points)

Routine problems:



a) (2 points) $\langle 3,1,4\rangle \cdot \langle 5,9,2\rangle$.

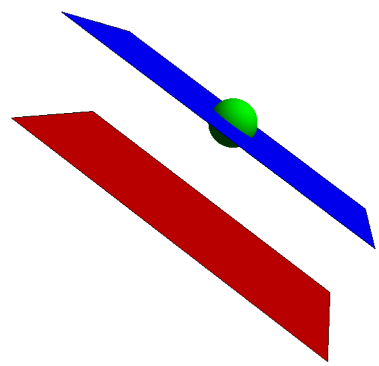
b) (2 points) $\langle 5,5,5\rangle \times \langle 3,2,1\rangle$.

c) (2 points) $\langle 5,5,5\rangle \cdot \langle 3,2,1\rangle \times \langle 1,1,1\rangle$.

d) (2 points) $\vec{P}_{\vec{w}}(\vec{v})$ with $\vec{v} = \langle 1,1,1\rangle$ and $\vec{w} = \langle 1,2,1\rangle$.

e) (2 points) $\cos(\alpha)$ of angle between $\langle 1,1,1\rangle, \langle 4,3,1\rangle$.

Problem 4) (10 points)



When two uncharged metallic parallel plates are put close together, there is an attractive force between them which can be explained by quantum field theory only. In May 14, 2013, an article suggested to use this Casimir effect for microchip designs. (Source Nature: http://www.nature.com/ncomms/journal/v4/n5/full/nc

a) (3 points) Locate a point P on the plane $x + 2y + 2z = 4$.

b) (7 points) Find the distance d between the plane $x + 2y + 2z = 1$ and plane $x + 2y + 2z = 4$.

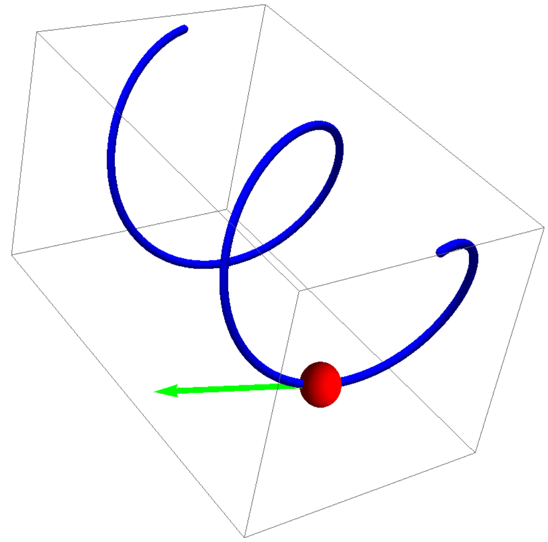
Problem 5) (10 points)

Given $\vec{r}(t) = \langle \sin(\pi t^2), t^2, \cos(\pi t^2) \rangle$.

a) (4 points) Find the arc length from $t = 0$ to $t = 1$.

b) (4 points) Find $\vec{r}\,'(t)$ and $\vec{r}\,''(t)$ at $t = 1$.

c) (2 point) What is the curvature at the point $t = 1$?



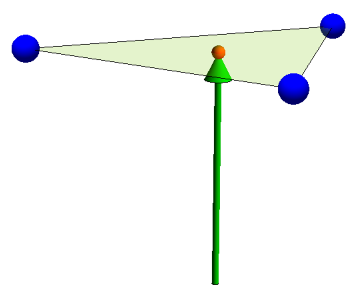
Problem 6) (10 points)

a) (2 points) Given three points $A = (1,1,1), B = (1,0,1), C = (0,1,1)$. Find the **center of mass** $M = (A + B + C)/3$.

b) (4 points) Find the equation $ax + by + cz = d$ of the plane through A, B, C.

c) (4 points) Find a parametrization of the line through M which is perpendicular to the triangle.

Now you can put a needle on this line and place the triangle on top of it. It will float in equilibrium.



Problem 7) (10 points)

The **Pound-Rebka experiment** was one of the latest tests for Einsteins theory of relativity. The experiment, which measures the gravitational red shift had been installed in the 22.5-meter-high Jefferson Tower at the Harvard physics department just behind the Science Center. To celebrate this achievement, we climb to the top of the Jefferson tower and throw a stone from $(0, 0, 25)$ towards the Science Center with initial velocity is $\langle 10, 0, 20 \rangle$. Where does the stone hit the ground, if the acceleration is $\langle 0, 0, -10 \rangle$?

Problem 8) (10 points)

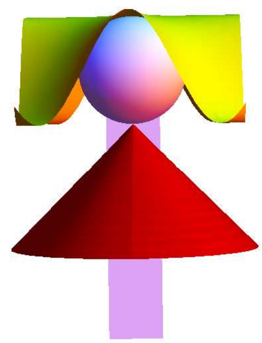
We build an action figure for a future MMOG "World of Math21a". One of the figures is composed of four parametric surfaces. Parametrize these surfaces in the form $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$. For example, to parametrize a cylinder $x^2 + y^2 = 1$, you would get $\vec{r}(u,v) = \langle \cos(u), \sin(u), v \rangle$. You do not have to give bounds for the parameters.

a) (3 points) sphere $(x-1)^2 + (y-2)^2 + z^2 = 9$.

b) (2 points) plane through $(0,0,0), (1,0,0), (1,1,1)$.

c) (3 points) cone $x^2 + z^2 = y^2$.

d) (2 points) graph of $f(x,y) = \cos(xy)$.



Problem 9) (10 points)

An asteroid moves on a curve $\vec{r}(t) = \langle t, t^2, t^4 \rangle$.

a) (5 points) Find the area of the triangle with vertices $A = \vec{r}(-1), B = \vec{r}(1)$ and $C = \vec{r}(0)$.

b) (5 points) To find, whether the point $D = \vec{r}(2) = \langle 2, 4, 16 \rangle$ is on the plane, compute the volume of the parallelepiped spanned by the four points.