7/24/2014 SECOND HOURLY PRACTICE VI Maths 21a, O.Knill, Summer 2014

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Provide details to all computations except for problems 1-3.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F The function $f(x, y) = x + y$ has no critical points at all.

2) T F The gradient of f at a point (x_0, y_0, z_0) is orthogonal to the level surface of f which contains (x_0, y_0, z_0).

3) T F If $(2, 3)$ is a local maximum for the function f with discriminant $D > 0$, then $f_{xx}(2, 3) < 0$.

4) T F If f satisfies the partial differential equation $f_x + f_y = 0$ everywhere, then the discriminant D is zero at every critical point.

5) T F If $f(x, y)$ has a saddle point at $(0, 0)$, then $-f(x, y)$ has a saddle point at $(0, 0)$.

6) T F The value of the function $f(x, y) = xy$ at $(x, y) = (3.1, 5.2)$ can be estimated as $15 + 0.5 + 0.6$.

7) T F The chain rule tells that $\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$

8) T F The gradient of f at a point (x_0, y_0) is parallel to the level curve of f which contains (x_0, y_0).

9) T F If $(1, 1)$ is a critical point of $f(x, y)$ and is not a critical point of $g(x, y)$ then $(1, 1)$ can not be a critical point of f under the constraint g.

10) T F If an airship always moves in the direction opposite to the gradient of the pressure, then the pressure momentarily decreases.

11) T F At points, where the velocity of a curve is zero, the curvature is zero.

12) T F If D is the discriminant at a critical point and $D < 0$ then $f_{xy} \leq 0$.

13) T F If $(0, 0)$ is a critical point of $f(x, y)$ and $f_{xx}(0, 0) > 0$ and $f_{yy}(0, 0) > 0$, then $(0, 0)$ can not be a local minimum.

14) T F The tangent plane to the graph $f(x, y) = z$ at a point (x_0, y_0) is parallel to $\langle 0, 0, 1\rangle$ if (x_0, y_0) is a critical point.

15) T F The directional derivative $D_{\vec{v}} f$ is a vector perpendicular to $\vec{v}$.

16) T F The integral $\int_0^1 \int_0^1 x^2 + y^2 \, dxdy$ is the volume of the solid bounded by the 5 planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and the paraboloid $z = x^2 + y^2$.

17) T F The directional derivative $D_v f(1, 1)$ is zero if v is a unit vector tangent to the level curve of f which goes through $(1, 1)$.

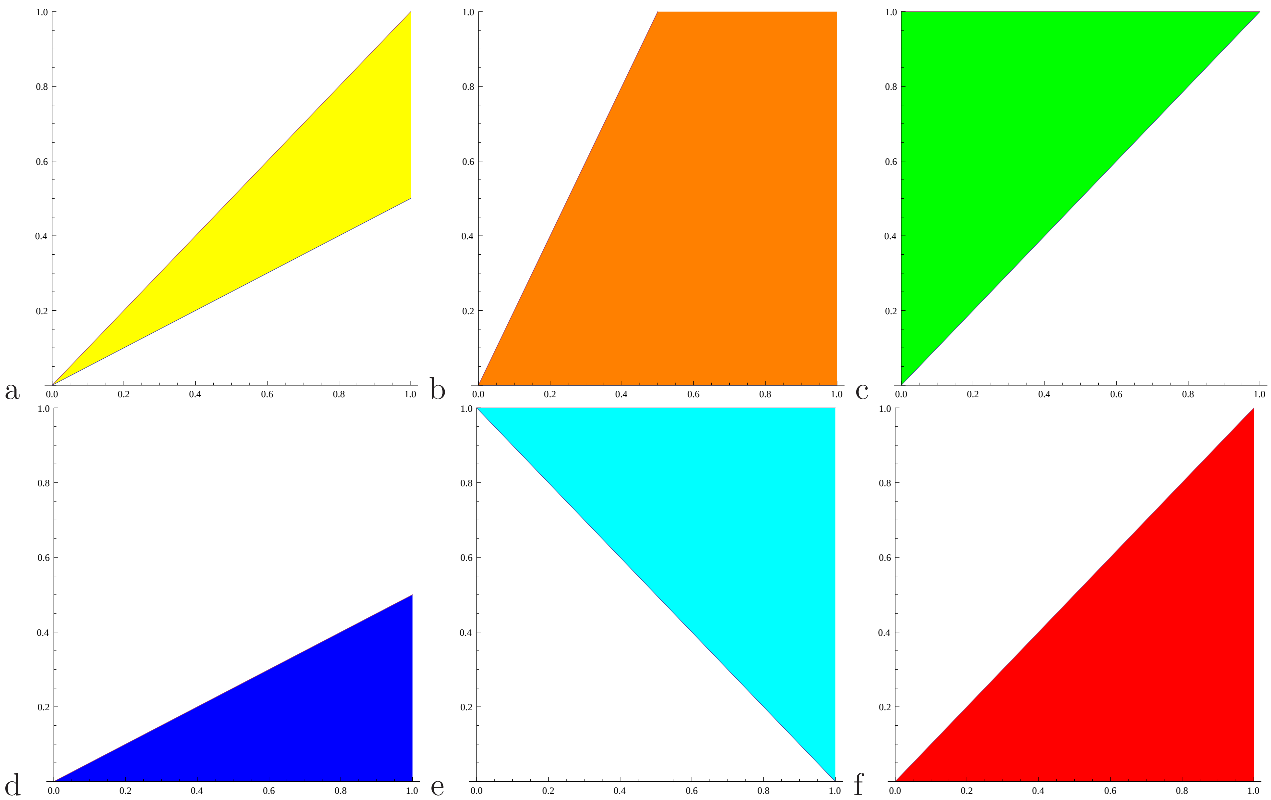
18) T F If (a, b) is a maximum of $f(x, y)$ under the constraint $g(x, y) = 0$, then the Lagrange multiplier λ there has the same sign as the discriminant $D = f_{xx} f_{yy} - f_{xy}^2$ at (a, b).

19) T F The surface area of a surface is independent of the parametrization

20) T F The double integral $\int_0^1 \int_0^1 xy \;\; dxdy$ is in polar coordinates the integral $\int_0^1 \int_0^1 r^2 \cos(\theta) \sin(\theta) \; r \; drd\theta$.

Problem 2) (10 points)

Match the regions with the corresponding double integrals



Enter a,b,c,d,e or f	Integral of Function $f(x,y)$
	$\int_0^1 \int_{x/2}^{x} f(x,y)\ dydx$
	$\int_0^1 \int_0^{y} f(x,y)\ dxdy$
	$\int_0^1 \int_0^{x/2} f(x,y)\ dydx$
	$\int_0^1 \int_{y/2}^{1} f(x,y)\ dxdy$
	$\int_0^1 \int_0^{x} f(x,y)\ dydx$
	$\int_0^1 \int_{1-x}^{1} f(x,y)\ dydx$

Problem 3) (10 points)

a) (4 points) Find all the critical points of the function $f(x, y) = xy$ in the interior of the elliptic domain

$$x^2 + \frac{1}{4}y^2 < 1 .$$

and decide for each point whether it is a maximum, a minimum or a saddle point.

b) (4 points) Find the extrema of f on the boundary

$$x^2 + \frac{1}{4}y^2 = 1 .$$

of the same domain.

c) (2 points) What is the global maximum and minimum of f on $x^2 + \frac{1}{4}y^2 \leq 1$.

Problem 4) (10 points)

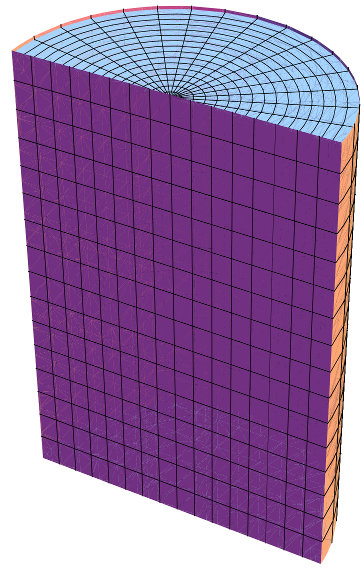
Oliver rides his bike along streets in the Massachusetts. Since the streets can be quite bumpy, he tries to avoid critical points which are maxima (bumps) or minima (potholes) but aims to drive over saddle points (mountain passes). Assume the street is the graph of the function

$$f(x, y) = \frac{x^4}{4} + \frac{y^4}{4} - \frac{x^2}{2} + \frac{y^2}{2} .$$

List all critical points and classify them as local maxima, local minima and saddle points.

Problem 5) (10 points)

Its amazing how many new energy drinks pop up every year and each tries to be original. Beside the concept like "water from Norway" or "vitamin drinks" or "taurin", "coffeine bomb", it needs also a catchy name "pink bull", "muscle milk" etc, and a fancy shape. Summer school is always a good time to come up with business ideas. We launch in this midterm an energy drink called "sweet tooth" which contains a insanely strong "caffeine, taurin, ginseng, isotonic vitamin" combination. But what really makes it stand out from the crowd is its shape: its half a cylinder of radius x and height y. Its only for tough guys or girls although: one first has to bite off a corner of the can with the teeth, in order to drink it ...



For a fixed volume $y\pi x^2/2 = \pi/2$, we want to minimize the material cost $\pi xy + \pi x^2 + 2xy$. In other words, we want to minimize the function

$$f(x, y) = (2 + \pi)xy + \pi x^2$$

under the constraint

$$g(x, y) = yx^2 = 1 \ .$$

Solve it with Lagrange method!

Problem 6) (10 points)

Find the arc length of the parameterized curve

$$\vec{r}(t) = \langle t^3 + 3, 2 + \cos(t^3), 3 + \sin(t^3) \rangle$$

from $t = 0$ to $t = \pi$.

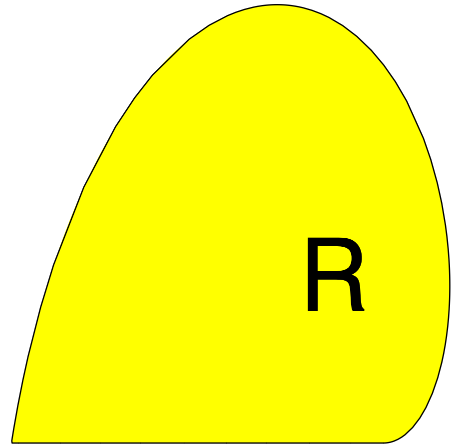
Problem 7) (10 points)

Estimate $f(x, y) = \sqrt{x^3 + y^3}$ for $x = 1.001, y = 2.01$ by linear approximation.

Problem 8) (10 points)

A region R in the xy-plane is given in polar coordinates by $r(\theta) \leq \theta$ for $\theta \in [0, \pi]$. You see the region in the picture to the right. Evaluate the double integral

$$\iint_R \frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}(\pi - \sqrt{x^2+y^2})} \, dx \, dy \, .$$



Problem 9) (10 points)

a) (5 points) Find the tangent plane to the surface $xy + x^2y + zx^2 = 20$ at the point $(2, 2, 2)$.
b) (5 points) Find the tangent line to the curve $x^2y^2 - xy + 3x = 3$ at the point $(1, 1)$.

Problem 10) (10 points)

Find the tangent plane to the surface

$$\sin(x+y) - \cos(z-x) + \sin(y) = -1$$

at the point $(0, \pi, 0)$.