7/24/2014 SECOND HOURLY PRACTICE II Maths 21a, O.Knill, Summer 2014

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Provide details to all computations except for problems 1-3.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points). No justifications needed.

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F The partial differential equation $u_{tt} = u_{xx}$ is an example of a heat equation.

2) T F The level curve $f = 0$ of $f(x, y) = x^2 - y^2$ intersects with the unit circle in 4 points.

3) T F The critical points of D are called the discriminants of $f(x, y)$.

4) T F The relation $f_{xyx}(x, y) = f_{yxx}(x, y)$ holds for all points (x, y) if f is the function $f(x, y) = y^9 \sin(\sin(\cos(\sin(yx^4))))$.

5) T F The tangent plane to the surface $x^2 + y^2 + z^2 = 1$ at the point $(1, 0, 0)$ is $x = 1$.

6) T F Fubini's theorem implies that for any function $f(x, y)$ of two variables, we have $\int_0^1 \int_3^4 f(x, y)\, dxdy = \int_0^1 \int_3^4 f(x, y)\, dydx$.

7) T F If a function $f(x, y)$ has a saddle point at $(0, 0)$ then $D_{\vec{v}} f(0, 0)$ takes positive and negative values at $(0, 0)$.

8) T F The integral $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1\, dydx$ is the area of the unit disc.

9) T F There is a function $f(x, y)$ for which the linearization at $(0, 0)$ is $L(x, y) = 1 + x^2 + 2y$.

10) T F The height of the Mont Blanc is $f(x, y) = 4810 - 2x^2 - y^2$. At height 4810 meters, the directional derivative in the direction $(1, 0)$ is negative.

11) T F If $\vec{r}(t)$ is a curve on the hyperbolic paraboloid $g(x, y, z) = x^2 - y^2 + z = 10$ then $\nabla g(\vec{r}(t)) \cdot \vec{r}\,'(t) = 0$.

12) T F If the length $|\nabla f(0, 0)|$ of the gradient is equal to 1, then there is a direction for which the slope of the graph of f at $(0, 0)$ is 1.

13) T F If $f(x, y)$ has a local minimum at $(0, 0)$, then the discriminant satisfies $D > 0$ and furthermore $f_{xx} < 0$.

14) T F $\int_0^1 \int_0^x f(x, y)\, dydx = \int_0^1 \int_0^y f(x, y)\, dydx$.

15) T F The area of the half disk of radius L in the half plane above the x-axes is $\int_0^\pi \int_0^L r\, drd\theta$.

16) T F If $f_{xy}(0, 0) = 0$, then f has a local maximum or a local minimum at $(0, 0)$.

17) T F The gradient vector $\nabla f(x, y)$ is a vector in space perpendicular to the graph of $f(x, y)$.

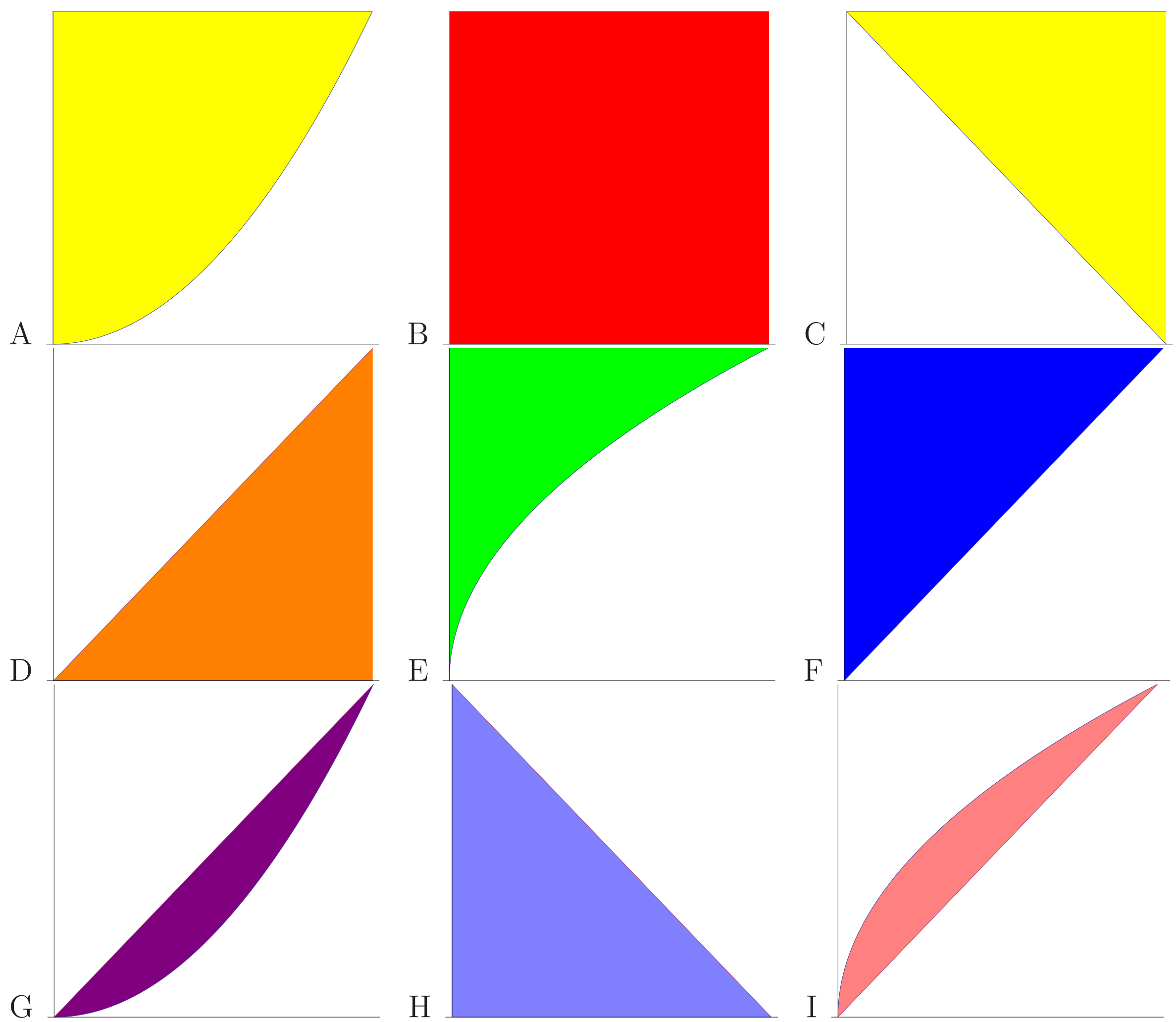
18) T F The surface area of the Gabriel trumpet $r = 1/z, 1 \leq r < \infty$ is finite because the volume of the inside is finite.

19) T F If the directional derivative at a point $(0, 0)$ is known in the direction $\langle 1, 1\rangle/\sqrt{2}$ and $\langle 1, 2\rangle/\sqrt{5}$ then the gradient $\nabla f(0, 0)$ is known.

20) T F If $f_x(x, y) = f_y(x, y)$ then every critical point (x_0, y_0) of f has zero discriminant $D = 0$.

Problem 2) (10 points) No justifications are needed

(10 points) Match the regions with the double integrals.



A-I	Integral
	$\int_0^2 \int_{x^2/2}^2 f(x,y)\, dydx$
	$\int_0^2 \int_0^x f(x,y)\, dydx$
	$\int_0^2 \int_0^2 f(x,y)\, dydx$

A-I	Integral
	$\int_0^2 \int_0^{y^2/2} f(x,y)\, dxdy$
	$\int_0^2 \int_0^y f(x,y)\, dxdy$
	$\int_0^2 \int_{y^2/2}^y f(x,y)\, dxdy$

A-I	Integral
	$\int_0^2 \int_{x^2/2}^x f(x,y)\, dydx$
	$\int_0^2 \int_0^{2-x} f(x,y)\, dydx$
	$\int_0^2 \int_{2-x}^2 f(x,y)\, dydx$

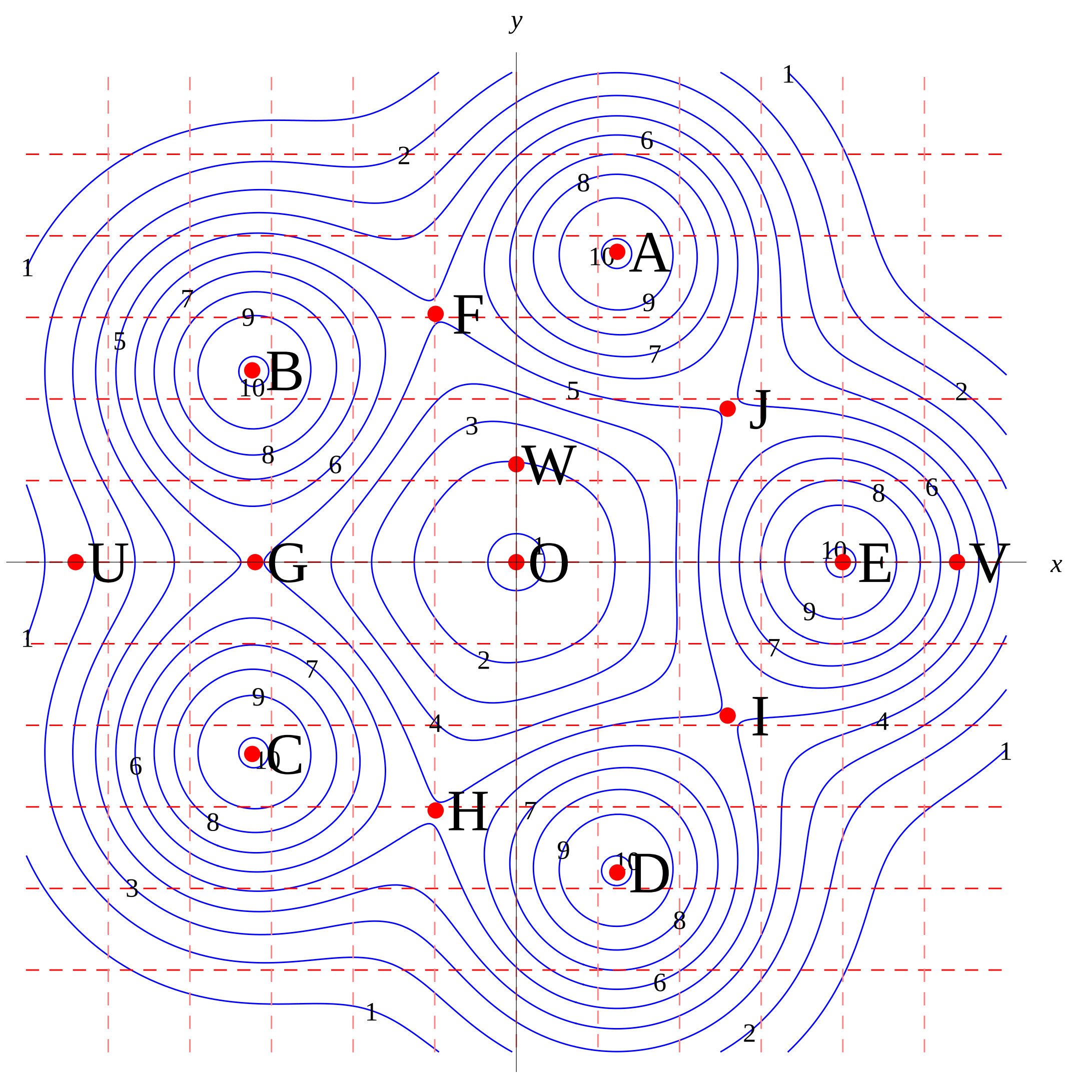
Problem 3) (10 points) (No justifications are needed.)

a) (6 points) A function $f(x, y)$ of two variables is shown as a contour map. Check what applies

	A	B	C	D	E	F	G	H	I	J	O	U	V	W
Local maximum														
Local minimum														
Saddle point														
Maximal steepness among A-W														
$f_x = 0, f_y \neq 0$.														
$f_y = 0, f_x \neq 0$.														

b) (4 points) Answer the following 4 questions.

What is the maximal height f reached when walking straight from U to V?	
What is the minimal height f reached when walking straight from U to V?	
There is a path from G to F on which height f is constant. True or False?	
There is a path from A to B on which height f is constant. True or False?	



Problem 4) (10 points)

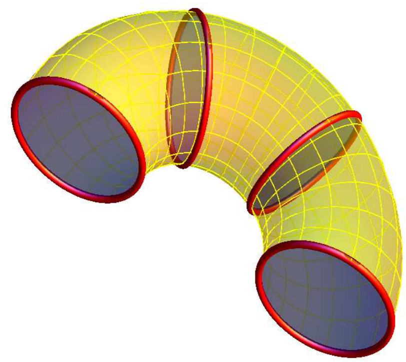
Maximize the volume of the half torus

$$f(x, y) = \pi^2 x^2 y$$

if the surface area of the torus including two disc dividers

$$g(x, y) = 4\pi x^2 + 2\pi^2 xy = \pi$$

is fixed.



Problem 5) (10 points)

Find all the critical points of the function

$$f(x, y) = x^3 - 3x + 1 + y^3 - 12y$$

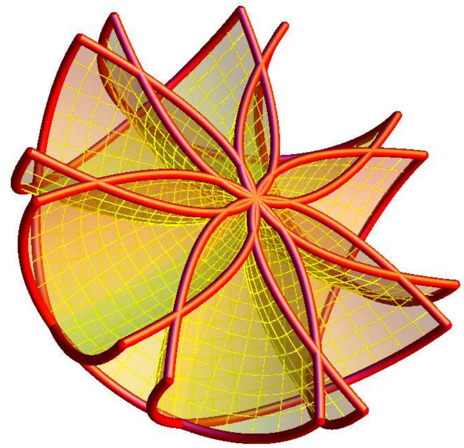
and classify them. Also note whether any global maxima or minima are present.

Problem 6) (10 points)

For an art project, we build a flower with 7 petals in which each petal is a paraboloid parametrized by

$$\vec{r}(u,v) = \langle u\cos(v), u\sin(v), u^2/2\rangle$$

with $0 \leq v \leq \pi$ and $0 \leq u \leq 1$. Find the surface area of the flower. You have to find the area A of one petal and multiply by 7.



Problem 7) (10 points)

a) (5 points) Estimate $f(0.003, 0.98) = \sin(0.003) \cdot 0.98^5 + 3 \cdot 0.98$ by linear approximation.

b) (3 points) Find $D_{\vec{v}} f(0,1)$ if $\vec{v} = \langle 1,1\rangle/\sqrt{2}$.

c) (2 points) If $\vec{r}(t) = \langle t, 1-t\rangle$ is a path, express

$$\frac{d}{dt} f(\vec{r}(t))$$

at $t = 0$ as a dot product of a vector with the vector $\vec{r}\,'(0) = \langle 1, -1\rangle$.

Problem 8) (10 points)

a) (5 points) Find the tangent line to $x + 2y^2 = 3$ at the point $P = (1,1)$.

b) (5 points) Find the tangent plane to $x^2 - y^2 + z^2 = 4$ at the point $(1,1,2)$.

Problem 9) (10 points)

a) (6 points) Integrate

$$\int_0^{\pi/2} \int_x^{\pi/2} \frac{\cos(y)}{y} \; dydx$$

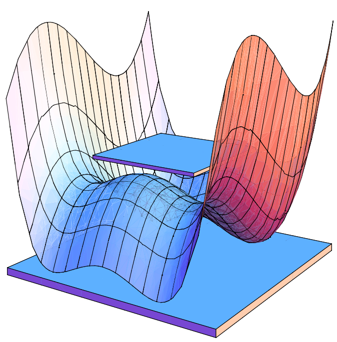
b) (4 points) Find the moment of inertia

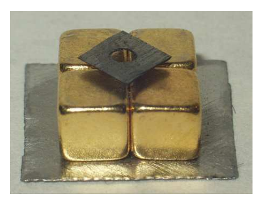
$$\iint_R (x^2 + y^2) \; dydx \; ,$$

where R is the ring $1 \leq x^2 + y^2 \leq 9$.

Problem 10) (10 points)

Oliver got a diagmagnetic kit, where strong magnets produce a force field in which pyrolytic graphic flots. The gravitational field produces a well of the form $f(x,y) = x^4 + y^3 - 2x^2 - 3y$. Find all critical points of this function and classify them. Is there a global minimum?





Right picture credit: Wikipedia.