7/10/2014 FIRST HOURLY Practice II **Maths 21a, O.Knill, Summer 2014**

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Except for problems 1-2, give details.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have exactly 90 minutes to complete your work.

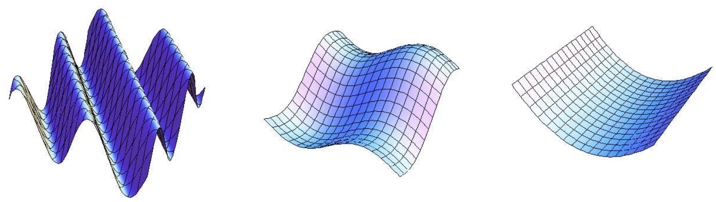
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

1)	T	F	$\langle 1,1,1\rangle \times \langle 1,1,1\rangle = 3$.
2)	T	F	The unit normal vector $\vec{N}$ is always perpendicular to the unit tangent vector $\vec{T}$.
3)	T	F	The point $(x,y,z) = (1,1,-\sqrt{2})$ is in spherical coordinates given by $(\rho,\phi,\theta) = (2,\pi/4,\pi/4)$.
4)	T	F	Since derivatives are related to smoothness of curves, **"Don't be a** d^3/dx^3**"** translates to **"Don't be so smooth"**.
5)	T	F	The curvature is a vector which points to the center of a circle which is most snug to the curve.
6)	T	F	The triple scalar product of the vectors $\vec{u},\vec{v},\vec{w}$ is equal to $\|u\|\|v\|\|w\|\sin(\alpha)\cos(\beta)$ where α is the angle between u and v and β is the angle between w and the $\vec{u}\times\vec{v}$.
7)	T	F	With the dot product we can determine the length of a vector as well as the angle between two unit vectors.
8)	T	F	The Cauchy-Schwartz inequality assures that $\langle 1,2,3\rangle \cdot \langle 2,3,4\rangle \leq \sqrt{1^2+2^2+3^2}\sqrt{2^2+3^2+4^2}$.
9)	T	F	The surface $x^2+y^2+z^2+2z=-3$ is a sphere.
10)	T	F	The arc length of $\langle \cos(t), t\rangle$ on the parameter interval $[0,1]$ is the same than the arc length of $\langle \cos(2t), 2t\rangle$ on the interval $[0,2]$.
11)	T	F	A plane and a line always intersect in space.
12)	T	F	The line $(x-1)=(y-2)=(z-3)$ hits the plane $x+y+z=1$ at a right angle.
13)	T	F	The planes $2x+6y+4z=1$ and $x+3y+2z=5$ are parallel.
14)	T	F	The parametrized curve $\langle \cos(t), \sin(t), \sin(t)\rangle$ is an ellipse, which is obtained by intersecting the plane $y=z$ with $x^2+y^2=1$.
15)	T	F	All hyperboloids and paraboloids are graphs $z=f(x,y)$.
16)	T	F	The vector $\langle 3/5, 1, 4/5\rangle$ is a unit vector.
17)	T	F	Two vectors $\vec{v}$ and $\vec{w}$ are parallel if $\vec{v}\cdot\vec{w}=0$.
18)	T	F	If $\vec{u},\vec{v}$ are two vectors, then $\vec{u},\vec{v},\vec{u}+\vec{v}$ span a parallelepiped of positive volume.
19)	T	F	The plane parametrized by $\vec{r}(t,s) = t\langle 1,0,0\rangle + s\langle 0,0,1\rangle$ is the same than $y=0$.
20)	T	F	The partial derivative of $f(x,y)=\sin(x^2y^2)$ with respect to x is equal to $2xy^2\cos(x^2y^2)$.

Problem 2) (10 points) No justifications are needed in this problem.

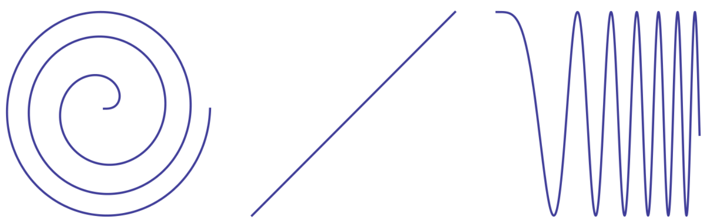
a) (2 points) Match the graphs of the functions $f(x, y)$. Enter O, if there is no match.



I II III

Function $f(x, y) =$	Enter O,I,II or III
$\sin(x + y)$	
$x^2 - y$	
x^2	
$y^4 - 2x^2$	
$\sin(x) + \sin(y)$	

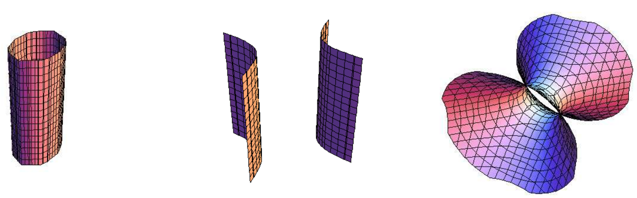
b) (3 points) Match the plane curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.



I II III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\vec{r}(t) = \langle \sqrt{t}\cos(t), \sqrt{t}\sin(t) \rangle$	
$\vec{r}(t) = \langle t^2, t \rangle$	
$\vec{r}(t) = \langle t, \cos(t^2) \rangle$	
$\vec{r}(t) = \langle 2t, 3t \rangle$	

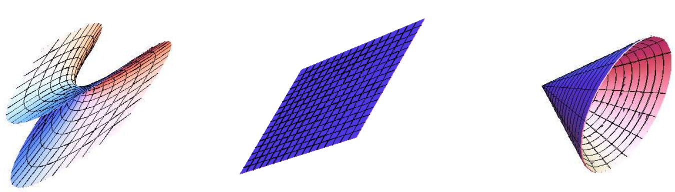
c) (2 points) Match functions g with level surface $g(x, y, z) = 1$. Enter O, if no match.



I II III

Function $g(x, y, z) = 1$	O, I,II,III
$g = 2x^2 + y^2 = 1$	
$g = xy = 1$	
$g = x^2 - y^2 + z^2 = 1$	
$g = -x^2 + y^2 - z^2 = 1$	

d) (3 points) Match the parametrization. Enter O, where no match.



I II III

$\vec{r}(s, t)$	O,I,II,III
$\langle t, s, t + s \rangle$	
$\langle t, t^2 - s^2, s \rangle$	
$\langle s, s\cos(t), s\sin(t) \rangle$	
$\langle \cos(t)\sin(s), \sin(t)\sin(s), \cos(s) \rangle$	

Problem 3) (10 points)

Routine problems:

a) (2 points) $\langle 1, 2, 3 \rangle \cdot \langle 3, 2, 1 \rangle$

b) (2 points) $\langle 1,2,3\rangle \times \langle 3,2,1\rangle$

c) (2 points) $\langle 1,2,3\rangle \cdot \langle 3,2,1\rangle \times \langle 2,1,3\rangle$

d) (2 points) The vector projection of $\langle 1,2,3\rangle$ onto the vector $\langle 3,2,1\rangle$

e) (2 points) The cosine of the angle between $\langle 1,2,3\rangle$ and $\langle 3,2,1\rangle$.

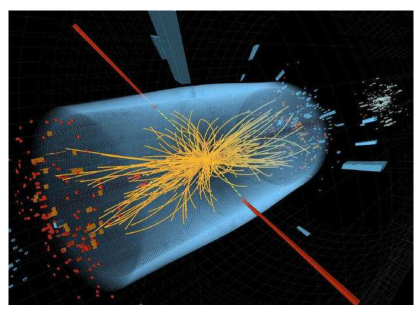
Problem 4) (10 points)

a) (7 points) Find the distance d between the plane $x + 2y + 2z = 1$ and and the point $P = (0,2,3)$.

b) (3 points) Find the radius of the circle obtained by intersecting the sphere of radius 5 centered at P with the plane.

Problem 5) (10 points)

a) (5 points) On July 4, the discovery of the Higgs boson was announced. A proton on the large hadron collider in Geneva is subject to a force $\vec{r}\,''(t) = \langle \cos(t), \sin(t), 0\rangle$ which is imposed on the particle by strong magnets. Assume it is at $\vec{r}(0) = \langle 1,1,10\rangle$ at time $t = 0$ and $\vec{r}\,'(0) = \langle 0,0,1\rangle$. Where is the particle at $t = \pi$?



b) (5 points) Write down the arc length integral of the parametrized curve $\vec{r}(t)$ for which $t \in [0, 2\pi]$.

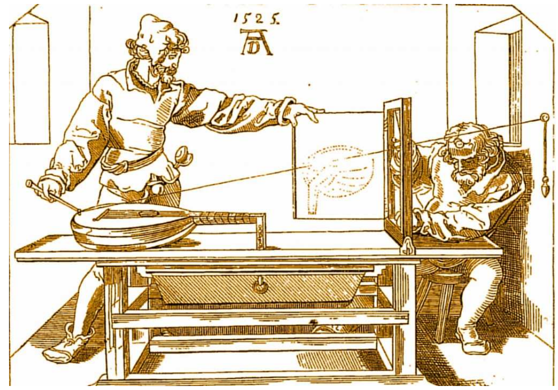
Problem 6) (10 points)

a) (5 points) Parametrize a line perpendicular to the plane $x - y + z = 5$ which is at time $t = 0$ at the point $(0,0,2)$.

b) (5 points) Intersect this line with the sphere $x^2 + y^2 + z^2 = 36$.

Problem 7) (10 points)

The plane $\Sigma : x + 2y + z = 1$ is the photographic plate and $O = (0, 1, 0)$ is the viewpoint. For a point like $P = (3, 4, 5)$ we form the line through O and P and intersect with the plane.



a) (5 points) Parametrize the line L as $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$.

b) (5 points) Intersect the line L with the plane Σ to get the image point on the plane.
Picture "Perspective machine" by Albrecht Dürer, 1525

Problem 8) (10 points)

Find the parametrizations $\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$. You can of course use other variables than u, v. For example, if $y = \sin(xz)$, then $\vec{r}(x, z) = \langle x, \sin(xz), z \rangle$.

a) (2 points) The hyperbolic paraboloid $z = x^2 - y^2$.

b) (2 points) The one sheeted paraboloid $x^2 + y^2 - z^2 = 1$.

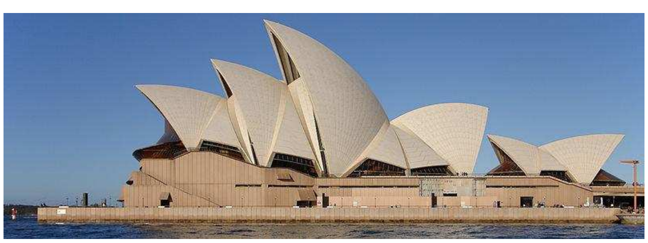
c) (2 points) The ellipsoid $(x - 1)^2 + y^2 + \frac{z^2}{4} = 1$

d) (2 points) The cone $x^2 + z^2 = y^2$.

e) (2 points) A roof the Sidney opera house in Sidney is parametrized as

$$\vec{r}(t, s) = \langle \cos(t) \sin(s), \cos(s), \sin(t) \sin(s) \rangle \ .$$

What surface is that?



Problem 9) (10 points)

a) (5 points) What is the area of the triangle through the points $A = (1, 1, 1)$ and $B = (0, 1, 0)$ and $C = (1, 2, 4)$.

b) (5 points) Find the volume of the prism which has the triangle T as base as well as a by $\vec{v} = \langle 0, 1, 1 \rangle$ translated triangle as roof.