8/5/2010 FINAL EXAM PRACTICE II **Maths 21a, O. Knill, Summer 2010**

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work.
- Do not detach pages from this exam packet or unstaple the packet.
- Please try to write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- Problems 1-3 do not require any justifications. For the rest of the problems you have to show your work. Even correct answers without derivation can not be given credit.
- You have 180 minutes time to complete your work.

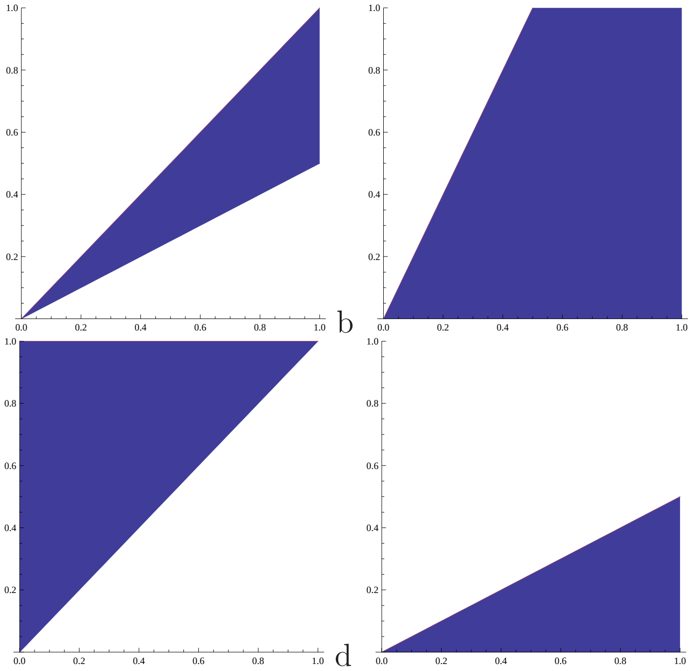
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
14		10
Total:		150

Problem 1) (20 points)

1)	T	F	The quadratic surface $x^2 + y - z^2 = -5$ is a hyperbolic paraboloid.
2)	T	F	There are vectors $\vec{u}$ and $\vec{v}$ such that $\|\vec{u} \times \vec{v}\| > \|\vec{u}\|\|\vec{v}\|$.
3)	T	F	$\int_0^{2\pi} \int_0^5 r \, d\theta \, dr$ is the area of a disc of radius 5.
4)	T	F	If a vector field $\vec{F}(x, y)$ satisfies $\text{curl}(F)(x, y) = Q_x - P_y = 0$ for all points (x, y) in the plane, then $\vec{F}$ is a gradient field.
5)	T	F	The jerk of a parameterized curve $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ is parallel to the acceleration if the curve $\vec{r}(t)$ is a line.
6)	T	F	The curvature of the curve $\vec{r}(t) = \langle 3\sin(t), 0, 3\cos(t) \rangle$ is twice the curvature of the curve $\vec{s}(t) = \langle 6 + 6\sin(t), 6\cos(t), 0 \rangle$.
7)	T	F	The curve $\vec{r}(t) = \langle \sin(t), t^2, \cos(t) \rangle$ for $t \in [0, 10\pi]$ is located on a cylinder.
8)	T	F	If a function $f(x, y)$ has the property that $f_x(x, y)$ is zero for all x, y, then f is the constant function.
9)	T	F	If the unit tangent vector $\vec{T}(t)$ of a curve $\vec{r}(t)$ is always parallel to a plane Σ, then the curve is contained in a plane parallel to Σ.
10)	T	F	If (x_0, y_0) is an extremum of $f(x, y)$ under the constraint $x^2 + y^2 = 1$, then the same point is an extremum of $10f(x, y)$ under the same constraint.
11)	T	F	At a critical point (x_0, y_0) of a function $f(x, y)$ for which $f_{xx}(x_0, y_0) > 0$, the critical point is always a minimum.
12)	T	F	If a vector field $\vec{F}(x, y)$ is a gradient field, and C is a closed curve which looks like a figure 8, then $\int_C \vec{F} \cdot d\vec{r}$ is zero.
13)	T	F	If C is part of a level curve of a function $f(x, y)$ and $\vec{F} = \langle f_x, f_y \rangle$ is the gradient field of f, then $\int_C \vec{F} \cdot d\vec{r} = 0$.
14)	T	F	The divergence of the gradient vector field $\vec{F}(x, y, z) = \nabla f(x, y, z)$ is always the zero function.
15)	T	F	The line integral of the vector field $\vec{F}(x, y, z) = \langle x, y, z \rangle$ along a line segment from $(0, 0, 0)$ to $(1, 1, 1)$ is $3/2$.
16)	T	F	The area of a region G can be expressed as a line integral along its boundary.
17)	T	F	The flux of the vector field $\vec{F}(x, y, z) = \langle x, y, -z \rangle$ through the boundary S of a solid ellipsoid E is equal to the volume the ellipsoid.
18)	T	F	If $\vec{F}$ is a vector field in space and S is a torus surface, then the flux of $\text{curl}(\vec{F})$ through S is 0.
19)	T	F	If the divergence and the curl of a vector field $\vec{F}$ are both zero, then it is a constant field.
20)	T	F	For any function f, the curl of $\vec{F} = \text{grad}(f)$ is the zero field $\langle 0, 0, 0 \rangle$.

Problem 2) (10 points)

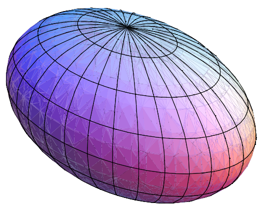
a) (4 points) Match the regions with the corresponding double integrals

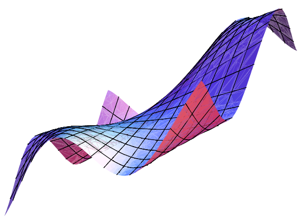


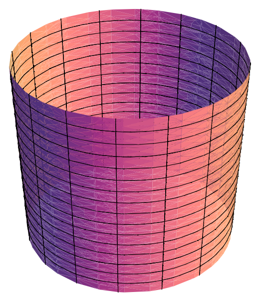
a b c d

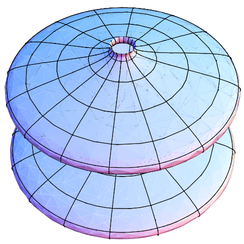
Enter a,b,c,d	Function
	$\int_0^1 \int_{x/2}^{x} f(x,y)\, dydx$
	$\int_0^1 \int_0^{y} f(x,y)\, dxdy$
	$\int_0^1 \int_0^{x/2} f(x,y)\, dydx$
	$\int_0^1 \int_{y/2}^{1} f(x,y)\, dxdy$

b) (6 points) Match the parametrized or implicit surfaces with their definitions


A


B


C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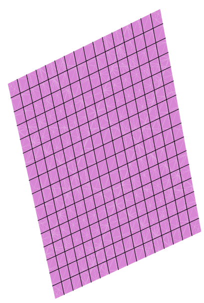
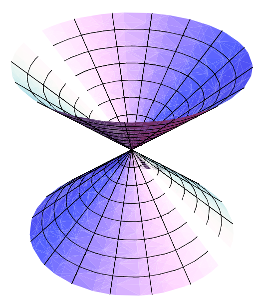
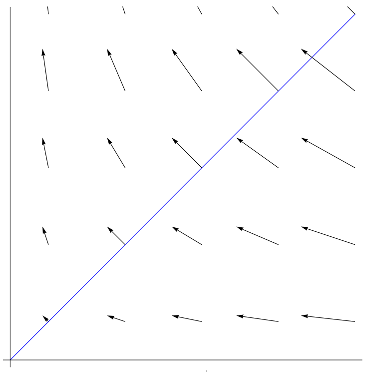

E


F

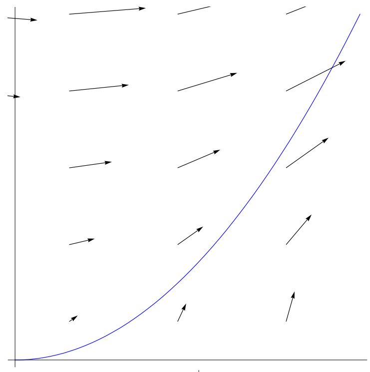
Enter A-F here	Function or parametrization
	$\vec{r}(u,v) = \langle \cos(u), \sin(u), v \rangle$
	$\vec{r}(u,v) = \langle u-v, u+2v, 2u+3v \rangle$
	$x^2 + y^2/3 + z^2/3 = 1$
	$\vec{r}(u,v) = \langle (\sin(v)+1)\cos(u), (\sin(v)+1)\sin(u), v \rangle$
	$z - x + \sin(xy) = 0$
	$x^2 + y^2 - z^2 = 0$

Problem 3) (10 points)

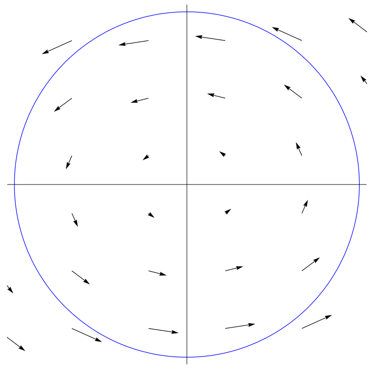
a) (4 points) Match the vector fields and curves with the corresponding line integral



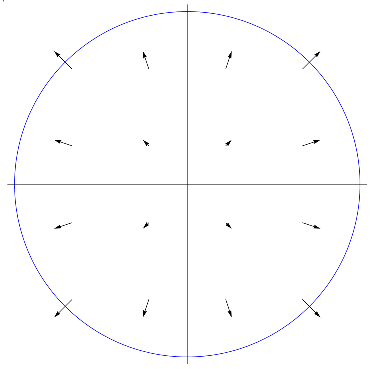
I



II



III



IV

Enter I,II,III,IV	Line integral
	$\int_0^{2\pi} \langle \cos(t), \sin(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle \; dt$
	$\int_0^{2\pi} \langle -t, t^2 \rangle \cdot \langle 1, 1 \rangle \; dt$
	$\int_0^{2\pi} \langle t^2, t \rangle \cdot \langle 1, 2t \rangle \; dt$
	$\int_0^{2\pi} \langle -3\sin(t), 3\cos(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle \; dt$

b) (6 points) Fill in from following choice: "arc length", "surface area", "chain rule", "volume of parallelepiped", "area of parallelogram", "line integral", "flux integral", "curvature".

Formula	Name of formula or rule or theorem
$\int \int_R \lvert \vec{r}_u \times \vec{r}_v \rvert \; dudv$	
$\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$	
$\int_a^b \lvert \vec{r}\,'(t) \rvert \; dt$	
$\frac{\lvert \vec{r}\,'(t) \times \vec{r}\,''(t) \rvert}{\lvert \vec{r}\,'(t) \rvert^3}$	
$\lvert \vec{u} \cdot (\vec{v} \times \vec{w}) \rvert$	
$\int_0^1 \int_0^1 \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v) \; dudv$	

Problem 4) (10 points)

Given the line $x - 1 = y - 2 = z - 3$ and the point $P = (8, 4, 5)$. Find the equation

$$ax + by + cz = d$$

of the plane which contains the line and the point.

Problem 5) (10 points)

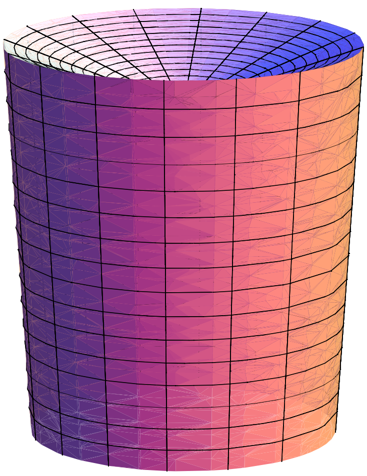
Find all the critical points of the function $f(x, y) = y^3 - 3y^2 + 4x + x^2 - 3$ and classify them by telling whether they are local maxima, local minima or saddle points.

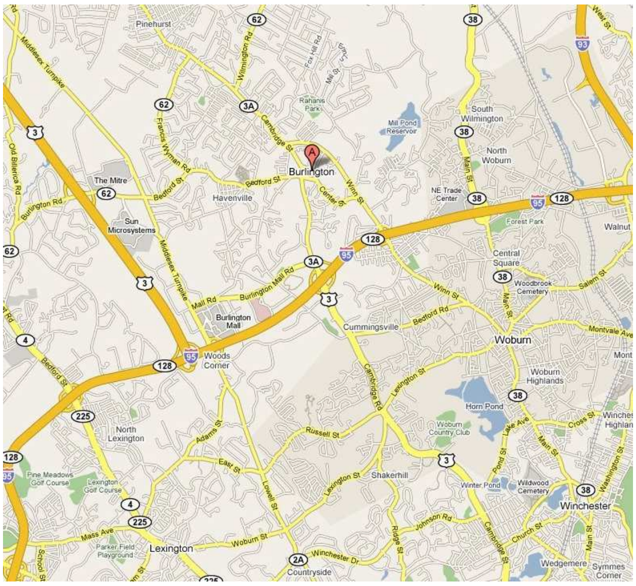
Problem 6) (10 points)

The hyperbolic paraboloid $x^2 - y^2 - 3z = 0$ contains the point $P = (1, 1, 0)$ and the point $Q = (3, 0, 3)$. Find the tangent planes to the surface at P and Q and find a parametrization $\vec{r}(t)$ of the line of intersection of these two planes.

Problem 7) (10 points)

A water reservoir in Burlington, MA (the map to the right is centered there) is bounded by a solid cylinder $x^2 + y^2 \leq 1$. It has as the roof the cone $x^2 + y^2 = (z - 6)^2$ and is bounded from below by the xy-plane $z = 0$. What is the volume of the reservoir?





Problem 8) (10 points)

Find the maxima and minima of the function $f(x, y) = x^2 - y^2$ on the parabola $x + y^2 = 1$ using the Lagrange multiplier method.

Problem 9) (10 points)

Compute the surface area of the surface $\vec{r}(u, v) = \langle u^3, v^3, u^3 - v^3 \rangle$ parametrized so that (u, v) is in the unit disc.

Problem 10) (10 points)

Evaluate the following double integral

$$\int_0^2 \int_{x/2}^1 \cos(y^2) \, dy \, dx \; .$$

Problem 11) (10 points)

Find the value of the line integral

$$\int_C \vec{F}(\vec{r}(t)) \; \cdot \vec{r}\,'(t) \; dt \; ,$$

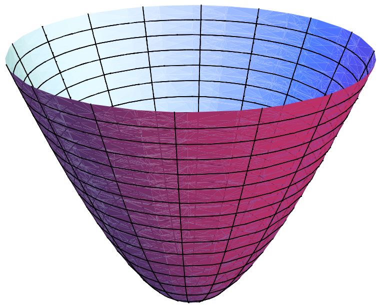
where $\vec{F}(x, y) = \langle y + \sin(\cos(x)), -2x \rangle$ and C is the boundary of the unit circle traversed in the counter clockwise direction.

Problem 12) (10 points)

Find the value of the flux integral

$$\int \int_S \text{curl}(\vec{F})(\vec{r}(u, v)) \; \cdot \vec{r}_u \times \vec{r}_v \; dudv$$

where $\vec{F}(x, y, z) = \langle -y, x, z \rangle$ and S is the part of the two-sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ which satisfies $1 < z < 2$ and which is oriented so that the normal vector points downwards on S.

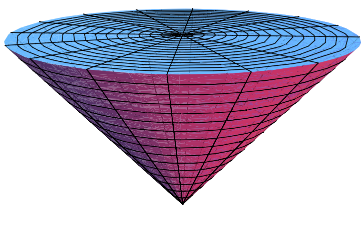


Problem 13) (10 points)

Let E be the solid which is bounded on the side by the cone $S_1 : x^2 + y^2 = z^2, 0 < z < 1$ and on top by the disc $S_2 = x^2 + y^2 \leq 1, z = 1$. Let $\vec{F}(x, y, z) = \langle 1 + 4x, 2 - 5y, 3 + 2z \rangle$. Find the value of the flux integral

$$\int_S \vec{F}(\vec{r}(u, v)) \cdot \vec{r}_u \times \vec{r}_v \, dudv ,$$

where S is the union of the two surfaces S_1 and S_2. The normal vector of S is oriented outwards on $S_1 \cup S_2$.



Problem 14) (10 points)

Find the volume of the solid piece of **cheese** bound by the cylinder $x^2 + y^2 = 1$, the planes $y - z = 0$ (bottom boundary) and $y + z = 0$ (top boundary) which is on the quadrant $x \geq 0$ and $y \leq 0$.