7/10/2008 SECOND HOURLY PRACTICE Maths 21a, O.Knill, Summer 2008

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
Total:		120

Problem 1) (20 points) No justifications are needed.

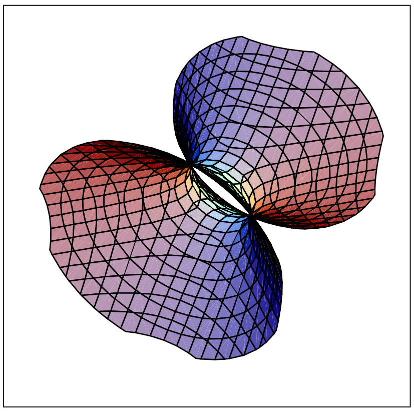
1)	T	F	The vector $\vec{v} = (1, 2, -4)$ is perpendicular to the plane $4x + 2y + 2z = 100$.
2)	T	F	With $\vec{i} = (1, 0, 0), \vec{j} = (0, 1, 0), \vec{k} = (0, 0, 1)$, the formula $(\vec{i} \times \vec{j}) \times (\vec{j} \times \vec{i}) = \vec{0}$ holds.
3)	T	F	The equations $x/5 = y/7 = z/8$ describe a line which contains the origin $(0, 0, 0)$.
4)	T	F	The vectors $\vec{u} = (3, -2, 1)$ and $\vec{OQ}$ with $O = (0, 0, 0)$ and $Q = (-6, 4, 2)$ are parallel.
5)	T	F	If $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ are perpendicular, then the vectors $\vec{u}$ and $\vec{v}$ have the same length.
6)	T	F	The two vectors $(2, 3, 0)$ and $(6, -4, 5)$ are orthogonal.
7)	T	F	For any two vectors $\vec{v}, \vec{w}$, one has $\vert\vec{v} + \vec{w}\vert^2 = \vert\vec{v}\vert^2 + \vert\vec{w}\vert^2$.
8)	T	F	The surface $x^2 - y^2 + z^2 = 1$ is called a one-sheeted hyperboloid.
9)	T	F	The set of points which have distance 2 from the x-axis is a cylinder.
10)	T	F	If in spherical coordinates a point is given by $(\rho, \theta, \phi) = (2, \pi/2, \pi/2)$, then its Euclidean coordinates is $(x, y, z) = (0, 2, 0)$.
11)	T	F	The triangle defined by the three points $(-1, 0, 2), (-4, 2, 1), (1, -1, 2)$ has a right angle.
12)	T	F	A surface which is given as $r = 2 + \sin(z)$ in cylindrical coordinates stays the same when we rotate it around the z axis.
13)	T	F	The length $\vert\vec{v} - \vec{w}\vert$ of the difference $\vec{v} - \vec{w}$ of two parallel vectors $\vec{v}, \vec{w}$ is always equal to the difference $\vert\vec{v}\vert - \vert\vec{w}\vert$ of the lengths of the vectors.
14)	T	F	The volume of a parallelepiped spanned by $(1, 0, 0), (0, 1, 0)$ and $(1, 1, 1)$ is equal to $1/3$.
15)	T	F	The equation $x^2 = y + z^2$ describes an elliptic paraboloid.
16)	T	F	In spherical coordinates, the equation $\cos(\theta) = \sin(\theta)$ is the plane $x-y = 0$.
17)	T	F	If $\vert\vec{v} \times \vec{w}\vert = 0$ then $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$ or $\vec{v} = \vec{w}$.
18)	T	F	The vector projection of the vector $(1, 1, 1)$ onto the vector $(0, 2, 0)$ is $(0, 1, 0)$.
19)	T	F	The point given in spherical coordinates as $(\rho, \theta, \phi) = (\sqrt{8}, 3\pi/2, \pi/2)$ is in Cartesian coordinates the point $(x, y, z) = (2, -2, 0)$.
20)	T	F	If $g(x, y, z) = 0$ is an implicit equation, then $\vec{r}(u, v) = (u, v, g(u, v, g(u, v, 1)))$ is a parametrization of the surface.

Total

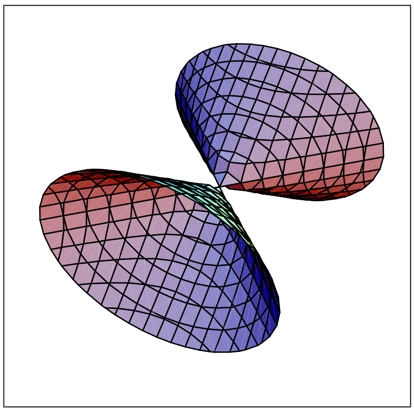
Problem 2) (10 points)

Match the equations $g(x,y,z) = d$ with the surfaces.

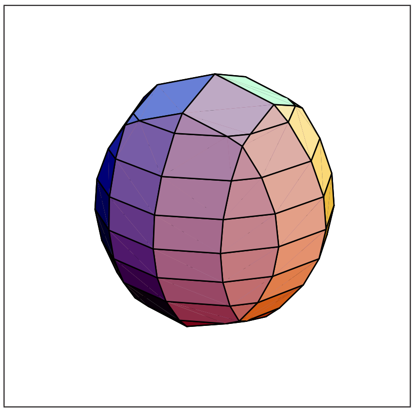
I



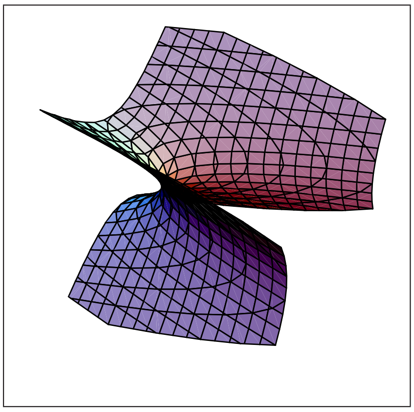
II



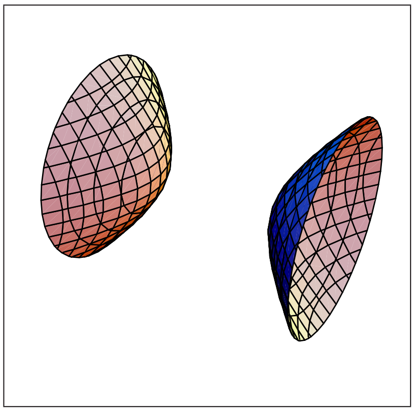
III



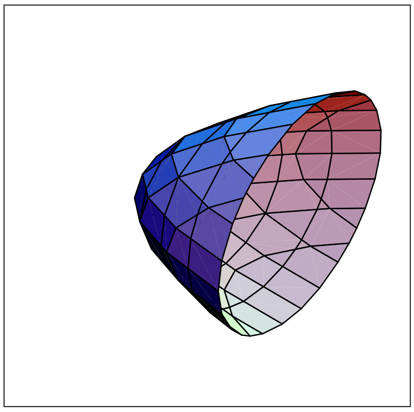
IV



V



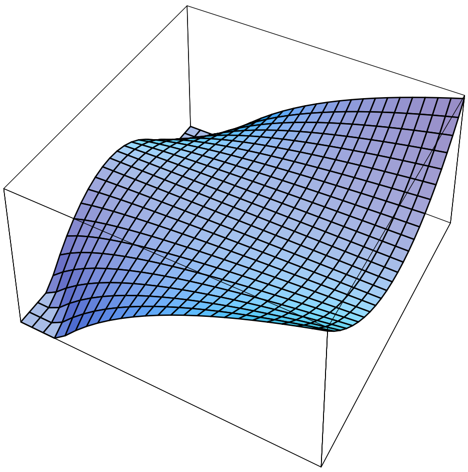
VI



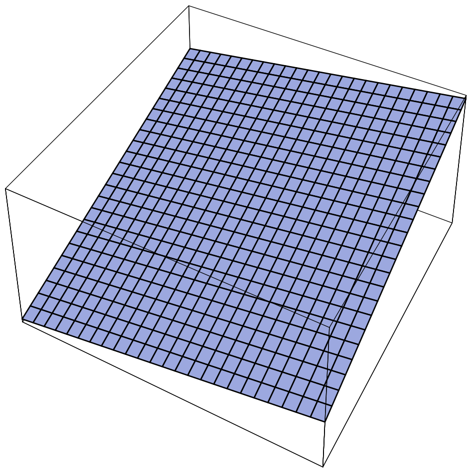
Enter I,II,III,IV,V,VI here	Equation
	$x^2 + 2z^2 - y^2 = 0$
	$y^2 + z^2 + 4x^2 = 1$
	$y^2 - z^2 - x^2 = -1$
	$x^2 - y^2 - z^2 = 1$
	$y^2 - z^2 - x = 1$
	$-y^2 - z^2 + x = 1$

Problem 3) (10 points)

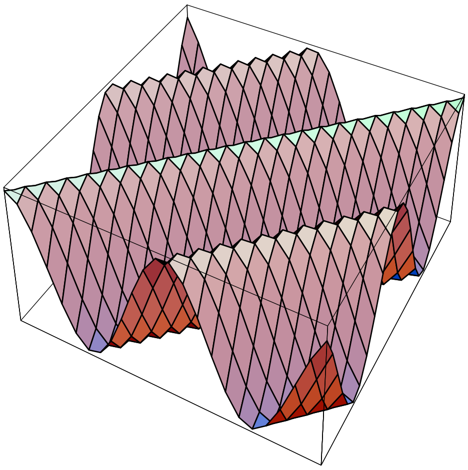
Match the equation with their graphs. No justifications are needed.



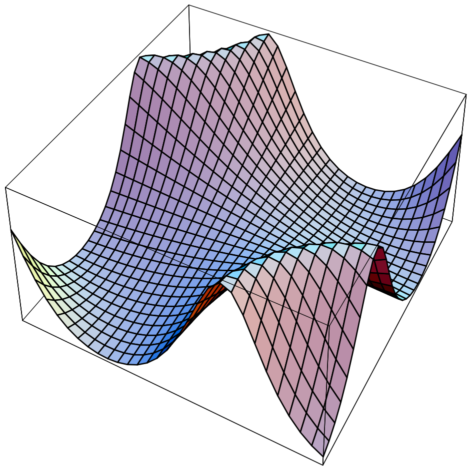
I



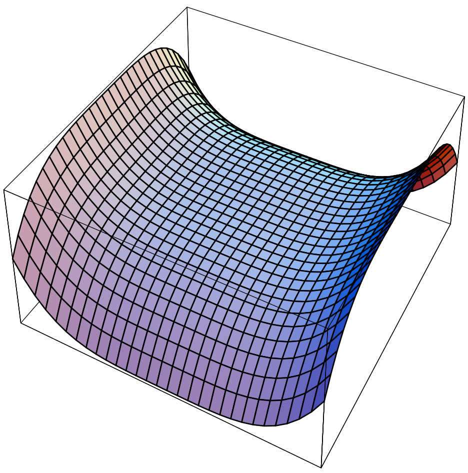
II



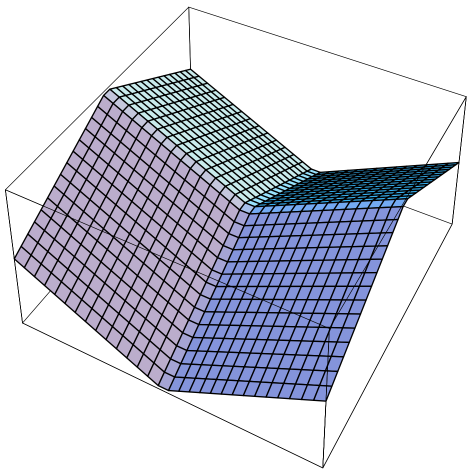
III



IV



V



VI

Enter I,II,III,IV,V or VI here	Equation
	$z = \|x\| - \|y\|$
	$z = y^2 \log(x)$
	$z = \cos(5(x-y))$
	$z = x + 2y$
	$z = x/(2+\sin(xy))$
	$z = x^4 - y^4$

Problem 4) (10 points)

Find the equation of the plane containing $A = (1,1,0)$ which is perpendicular to the planes

$$x + y + z = 1$$

and

$$x - y - z = 1 \ .$$

Problem 5) (10 points)

Find the distance between the point $P = (1,0,-1)$ and the line which contains the points $A = (1,1,1)$ and $B = (0,2,1)$.
To do so:

a) (4 points) Find first a parametrization $\vec{r}(t) = Q + t\vec{v}$ of the line.

b) (6 points) Now find the distance.

Problem 6) (10 points)

The angle between two planes is defined as the angle between the two normal vectors of the planes. Given the planes $x - z = 1$ and $y + z = 3$.

a) (2 point) Find the normal vector $\vec{v}$ to the first plane and the normal $\vec{w}$ to the second plane.

b) (2 points) Find the angle α between the two planes.

c) (2) points) Verify that $P = (4, 0, 3)$ and $Q = (1, 3, 0)$ are on the intersection of the two planes.

d) (2 points) Find a parametrization of the line of intersection of the two planes.

e) (2 points) Find the symmetric equation of the line of intersection of the two planes.

Problem 7) (10 points)

Find the implicit equation

$$ax + by + cz = d$$

of the plane which contains the line

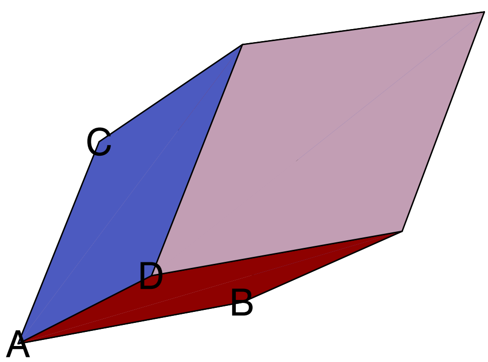
$$\vec{r}(t) = P + t\vec{v} = (-1, 1, 1) + t(3, 4, 5)$$

and the point $Q = (7, 7, 9)$.

Problem 8) (10 points)

a) (6 points) Find the surface area of the parallelepiped which contains the points $A = (0, 0, 0), B = (1, 1, 0), C = (0, 1, 1), D = (1, 0, 1)$.

b) (4 points) Find the volume of the solid.



Problem 9) (10 points)

a) (4 points) We define a scalar valued function which has as argument two vectors:

$$f(\vec{v},\vec{w}) = \frac{|\vec{v} \times \vec{w}|}{\vec{v} \cdot \vec{w}}$$

What is $f((1,0,0),(1,1,0))$?

b) (6 points) The function $f(\vec{v},\vec{w})$ turns out to be a function of the angle α between $\vec{v}$ and $\vec{w}$ only. What is this function?

Problem 10) (10 points)

The set of points P for which the distance from P to $A = (1,2,3)$ is equal to the distance from P to $B = (5,8,5)$ forms a plane S.

a) Find the equation $ax + by + cz = d$ of the plane S.

b) Find the distance from A to S.

Problem 11) (10 points)

Find the distance between the z-axis and the line

$$L : \frac{x+2}{4} = \frac{y-1}{3} = \frac{z}{2}.$$